THERMALIZATION

IN QCD

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CONCLUSION

- Thermalization occurs in perturbative QCD

thermalization time $\sim \dfrac{1}{\alpha^{13/5} Q_s} \downarrow$

temperature $\sim \alpha^{2/5} Q_s \nearrow$

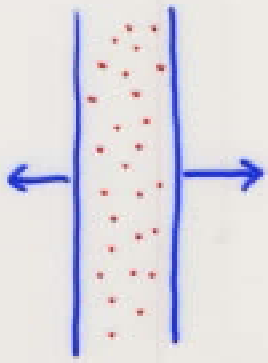
as $Q_s \nearrow$

Importance of inelastic processes

$2 \to 2$ DOES NOT EFFICIENTLY THERMALIZE

Consider the regime of one-dimensional expansion

$$t \ll R_{\rm nucl} \sim A^{1/3}\ \text{fm}$$



When $t \sim Q_s^{-1}$: gluons are produced, $p \sim Q_s$, occupation number $\sim 1/\alpha \Rightarrow N \sim Q_s^3/\alpha$

Subsequently:

$$N(t) = \frac{Q_s^3}{\alpha(Q_s t)}$$

Mean free time:

$$\tau = \frac{1}{\sigma N} = \frac{Q_s^2}{\alpha^2}\frac{\alpha Q_s t}{Q_s^3} = \frac{t}{\alpha}$$

or: thermalization time $\gg$ Hubble time.

worse at higher collision energy

WHY $2 \to 2$ IS INEFFICIENT

A lot of small angle scattering: $\sigma = \frac{\alpha^2}{q^2}$

q may be $\ll Q_s$, σ seems very large

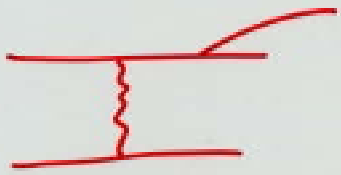
but each deflects the particle only by an angle $\theta \sim \frac{q}{Q_s}$.

Needs $\frac{1}{\theta^2} \sim \frac{Q_s^2}{q^2}$ small angle scatterings to change the distribution function $\Rightarrow$ relaxation is determined by the transport mean free time, $\sigma_{\rm tr} = \frac{\alpha^2}{q^2}\frac{q^2}{Q_s^2} = \frac{\alpha^2}{Q_s^2}$.

IMPORTANCE OF $2 \to 3$

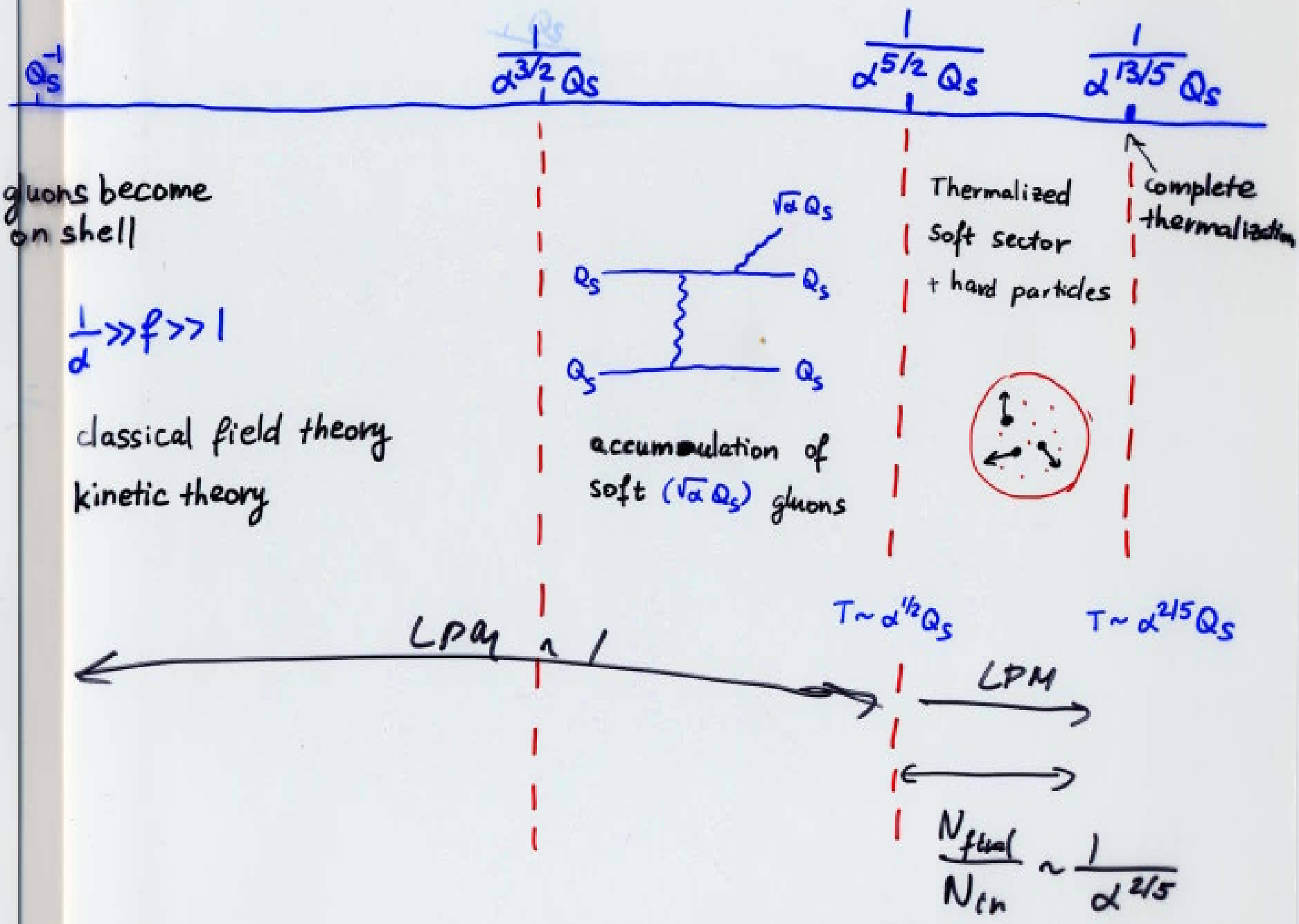
$\sigma_{2\to 3} \sim \alpha\sigma_{2\to 2}$, but does not requires multiple scatterings to change the distribution function $\Rightarrow$ very efficient:

$$\sigma_{2\to 3} \sim \frac{\alpha^3}{m_{\rm D}^2}$$



$m_{\rm D}$ = Debye screening, coming from primary hard gluons and secondary soft gluons emitted from $2 \to 3$

Timeline


Q_s^{-1}
$\frac{1}{\alpha^{3/2} Q_s}$
$\frac{1}{\alpha^{5/2} Q_s}$
$\frac{1}{\alpha^{13/5} Q_s}$
gluons become on shell
$\frac{1}{\alpha} \gg f \gg 1$
classical field theory
kinetic theory
$\sqrt{\alpha} Q_s$
Q_s
Q_s
Q_s
Q_s
accumulation of soft ($\sqrt{\alpha} Q_s$) gluons
Thermalized soft sector + hard particles
complete thermalization
$T \sim \alpha^{1/2} Q_s$
$T \sim \alpha^{2/5} Q_s$
LPM ~ 1
LPM
$\frac{N_{final}}{N_{in}} \sim \frac{1}{\alpha^{2/5}}$

From classical field eq to kinetic Boltzmann eq.

A.H. Mueller, DTS in progress

Classical field equation (schematically)

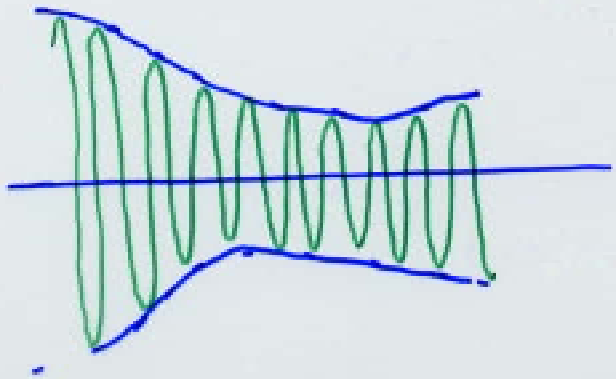
$$\partial^2 A + g A \partial A + g^2 A^3 = 0$$

when $\tau \sim Q_s^{-1}$ $\quad A \sim \frac{1}{g}$: strong nonlinearity

expansion $\Rightarrow$ $A \downarrow$ $\quad A \ll \frac{1}{g}$ when $\tau \gg Q_s^{-1}$

Regime of weak nonlinearity

$$A(t,\vec{x}) = \int d\vec{p} \; \underbrace{A(t,\vec{p})}_{\text{slow variation in } t} \; e^{-i\omega_p t + i\vec{p}\cdot\vec{x}} + \text{H.c.}$$



Eq for envelope : Boltzmann eq

Indirect way to derive Boltzmann eq

QFT $\longrightarrow$ Boltzmann eq with quantum statis

$f_p f_{p'} (1+f_q)(1+f_{q'}) - f_q f_{q'} (1+f_p)(1+f_{p'})$

$\hbar \to 0$ $\downarrow$ $\qquad\qquad$ $\downarrow$ $f_p \gg 1$

classical field theory $\xrightarrow{?}$ Boltzmann eq for classical waves

$f_p f_{p'} (f_q + f_{q'}) - f_q f_{q'} (f_p + f_{p'})$

$$f_p = \omega_p \langle |A(p)|^2 \rangle$$

Is there a way to directly go from classical field eqns to the Boltzmann eqn ?

One method : solve classical field eqn perturbatively
identify secular terms growing with time

does not have the generality one would like

The Martin - Siggia - Rose formalism

Martin, Siggia, Rose PRA, 8, 423 (1973)

Consider ϕ^4 theory

$$\partial_\mu^2 \phi + m^2 \phi + \lambda \phi^3 = 0$$

Generating functional:

$$Z = \int \mathcal{D}\phi \, \underbrace{\mathcal{D}\pi}_{\text{enforce field eq}} \, \exp\left\{ -i \int d^4x \left[\pi(\partial^2\phi + m^2\phi + \lambda\phi^3) + J_\pi \pi + J_\phi \phi \right] \right\}$$

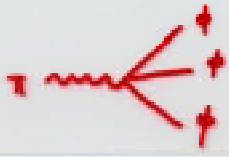
field doubling: classical limit of Schwinger - Keldysh

$\Rightarrow$ Feynman rules:

$$G_{\phi\pi}(x,y) = \langle \phi(x)\, \pi(y) \rangle$$
$$= G_R(x,y) = \int \frac{dp}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - m^2 + i\varepsilon \, \mathrm{sgn}\, p_0}$$

$$G_{\pi\phi}(x,y) = G_A(x,y)$$

$$G_{\phi\phi}(x,y) = \langle \phi(x)\, \phi(y) \rangle$$



$$-i\lambda$$

Schwinger - Dyson eqn :

$$-\begin{cases} (\partial_x^2 + m^2)\, G_{\phi\phi}(x,y) = \int dz \left[\Sigma_{\pi\phi}(x,z)\, G_{\phi\phi}(z,y) + \Sigma_{\pi\pi}(x,z)\, G_{\pi\phi}(z,y) \right] \\ (\partial_y^2 + m^2)\, G_{\phi\phi}(x,y) = \int dz \left[G_{\phi\phi}(x,z)\, \Sigma_{\phi\pi}(z,y) + G_{\phi\pi}(x,z)\, \Sigma_{\pi\pi}(z,y) \right] \end{cases}$$

and Wigner transform

$$G(x,y) = \int dp\; e^{-ip\cdot(x-y)}\; G\left(\frac{x+y}{2}, p\right)$$

assume slow variation in $X = \frac{x+y}{2}$, fast in $x-y$

(separation of scales)

$$p^\mu \frac{\partial}{\partial X^\mu} G_{\phi\phi}(X,p) = \underbrace{G_{\phi\phi}(\Sigma_{\pi\phi} - \Sigma_{\phi\pi}) + \Sigma_{\pi\pi}(G_{\pi\phi} - G_{\phi\pi})}_{\text{Collision term}}$$

with correct statistical factor

PERTURBATIVENESS

Heavy ion collisions at very high energy: can be described perturbative QCD.

$$Q_s \sim \begin{cases} 1 \text{ GeV at RHIC} \\ 2-3 \text{ GeV at LHC} \end{cases}$$

Q: does the system thermalize?

Pro: more gluons produced, collide more frequently.

Con:

- gluon distrubion initially far from equilibrium
- α_s smaller at higher energies

Goal: to have a consistent understanding of evolution when $\alpha_s \ll 1$, which is valid parametrically.

instanton