Contribution to Elliptic Flow from Particle Production in a Saturation Model

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Work done in collaboration with Kirill Tuchin

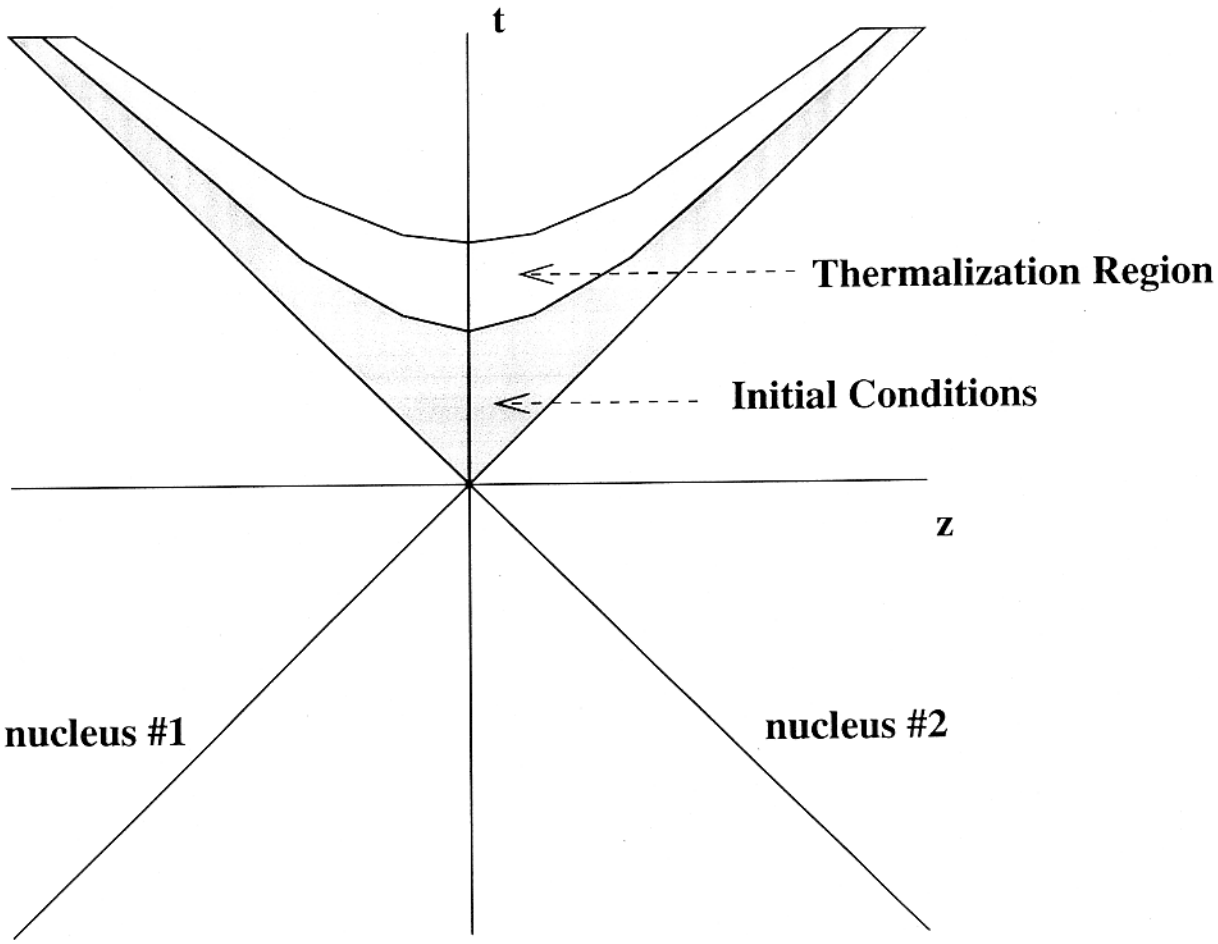


Figure 1: Space-time picture of a Heavy Ion Collision.

First few stages following a heavy ion collision:

1. Creation of gluons and quarks - construction of initial conditions.

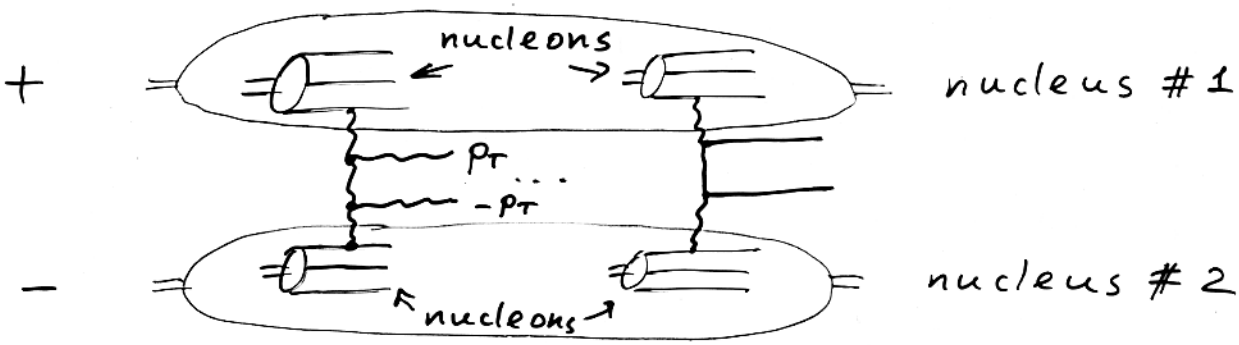
2. Possible thermalization of quarks and gluons.

Our goal is to explore the first and, partially, the stages of the collision and see how the recent results compare to RHIC data.

(I) Initial stage of the collision:

particles are produced.

(i) Collinear factorization ~ particles are produced in pairs:



⇝ valid only for high p_T, $p_T^2 \gg A^{1/3}\Lambda^2$

(higher twists are $\sim A^{1/3}\frac{\Lambda^2}{p_T^2}$ ⇒ need $\ll 1$)

neglect higher twists

⇝ high p_T of one gluon has to be compensated by high $-p_T$ of the other gluon

⇒ pairwise particle production

⇝ problem: $\frac{d\hat{\sigma}}{d^2p_T\, dy_1\, dy_2} \sim \frac{1}{p_T^4}$ ~ IR singular

$$\frac{dN}{dy} \sim \int \frac{dN}{d^2p_T\, dy}\, d^2p_T \sim \int \frac{d\sigma}{d^2p_T\, dy}\, d^2p_T \sim \int_{p_0} \frac{dp_T^2}{p_T^4} \sim \frac{1}{p_0^2}$$

some IR cutoff

⇒ # gluons is cutoff dependent!

(ii) Saturation approach:

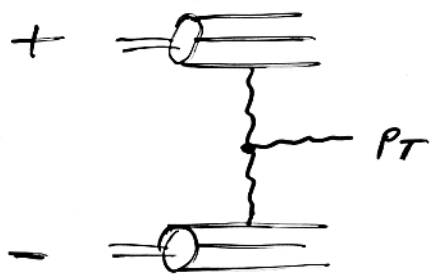
$\rightsquigarrow$ There is a large intrinsic scale in nuclei at high energies $\quad Q_s^2 \sim \dfrac{\frac{dN}{dy}}{\pi R^2} \gg \Lambda_{QCD}^2$

(# glue per unit transverse area)

$\alpha_s(Q_s) \ll 1 \Rightarrow$ everything is PERTURBATIVE!

$\rightsquigarrow$ do not need high external p_T anymore! $\Rightarrow$

$\Rightarrow$ can calculate from first principles:

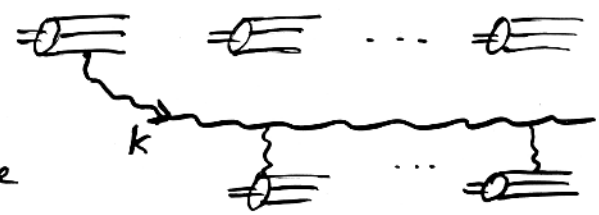


$\sim$ Lowest order diagram

no more pairwise production

$\Rightarrow$ can produce 1 gluon

$\rightsquigarrow$ IR problem:



coherence length of a glue

$$l_{coh} \sim \frac{k_+}{\underline{k}^2}$$

at some small $k_\perp$: $\quad l_{coh} \sim \dfrac{k_+}{k_{\perp min}^2} \sim R$ $\sim$ nuclear radius

$\Rightarrow$ gluon "sees" all nucleons (at given $\underline{b}$) & interacts

$\Rightarrow$ need to resum MULTIPLE RESCATTERINGS

McLerran–Venugopalan model '93 – '94

The initial conditions in heavy ion collisions are given by the classical gluon field.

I. First one has to find the classical gluon field of a single ultrarelativistic nucleus.

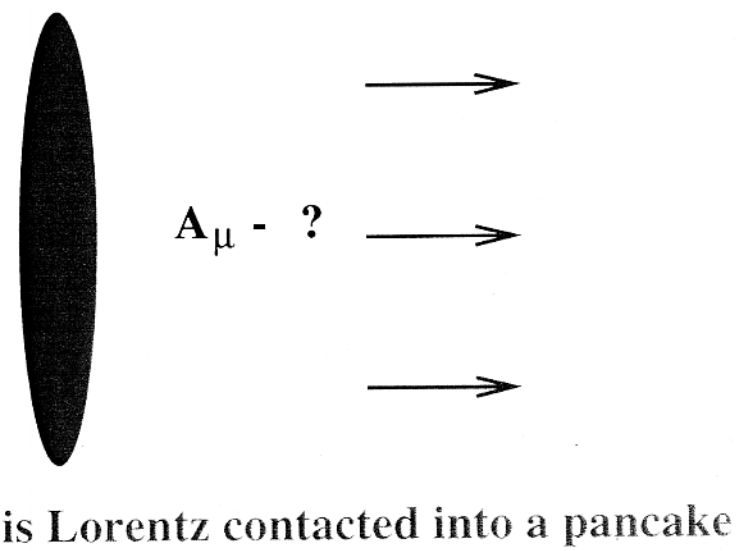


Figure 2: Classical gluon field of an ultrarelativistic nucleus.

One has to solve the non–linear Maxwell equations — Yang-Mills equations with the nucleus as a source carrying some color charge

$$\mathcal{D}_\nu F^{\mu\nu} = J^\mu$$

Yu. K. '96 ;

Jalilian-Marian, Kovner, McLerran, Weigert '96.

This field represents the of a large nucleus. One can look at the distribution of partons in this wave function.

$$k\frac{dN}{dk^2} \propto \langle A_\mu A_\mu \rangle$$

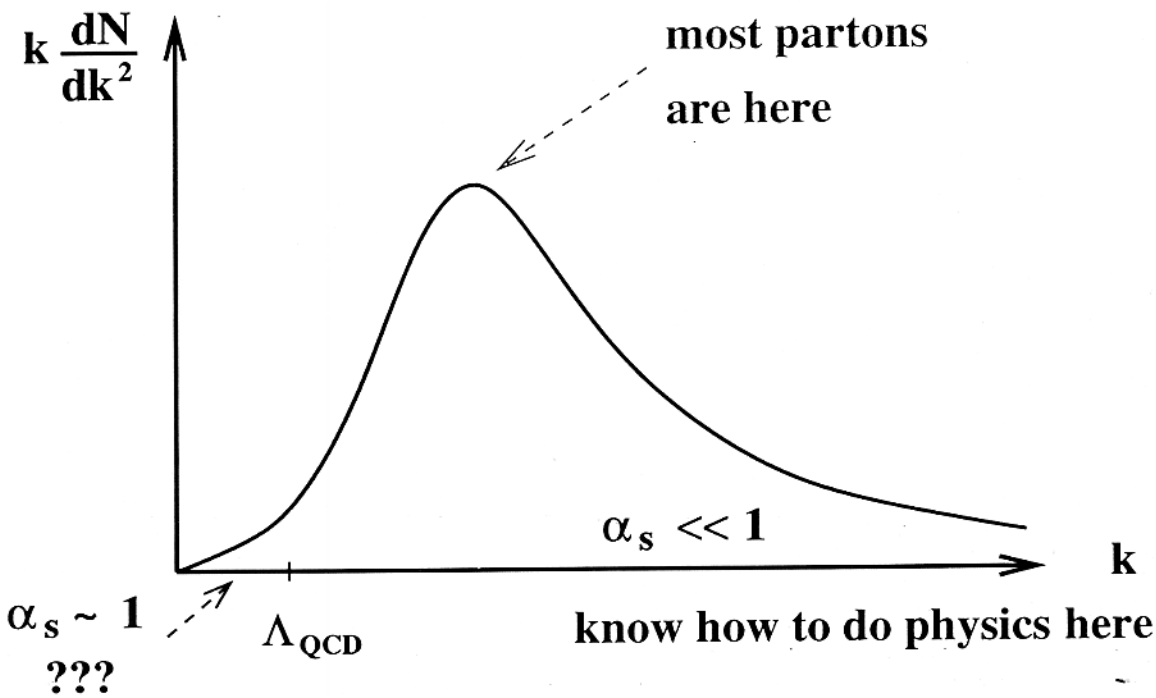
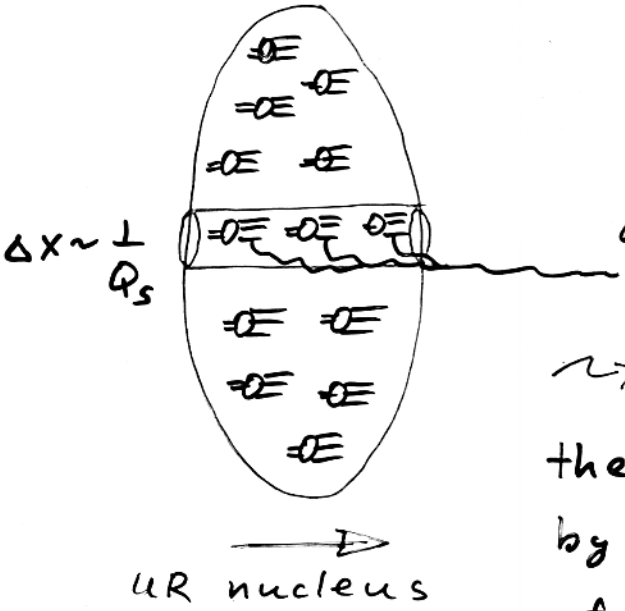


Figure 3: Distribution of partons in the nuclear wave function.

Most partons in the nucleus are distributed around the saturation momentum

$$Q_s^2 \sim A^{1/3} \Rightarrow Q_s^2 \gg \Lambda_{QCD}^2$$

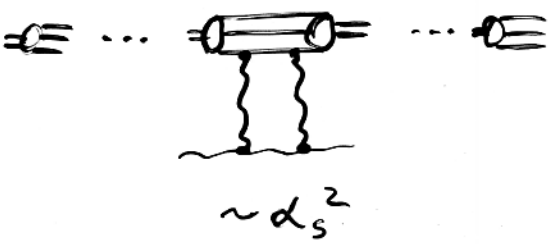
which can get big for a large nucleus (heavy ion).



$\rightsquigarrow$ typical momentum $\sim Q_s$

the field is produced coherently by all nucleons in a narrow "pipe" of radius $\sim \frac{1}{Q_s}$

$\rightsquigarrow$ all multiple rescatterings are included



$\sim A^{1/3}$ nucleons at given $\underline{b}$

$\Rightarrow$ resum $(\alpha_s^2 A^{1/3})^{\cdots}$

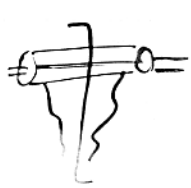
$\underset{\alpha_s(Q_s)}{\underset{\shortparallel}{\alpha_s}} \ll 1$, $A \gg 1$ (heavy ion) $\Rightarrow$ $\alpha_s^2 A^{1/3} \sim 1$

$\rightsquigarrow$ multiple rescatterings (higher twists) come in as $\dfrac{\alpha^2 A^{1/3} \Lambda^2}{p_T^2}$ $\sim$ they are important when

$$\alpha^2 A^{1/3} \left.\frac{\Lambda^2}{p_T^2}\right|_{p_T = Q_s} \sim 1 \quad \Rightarrow \quad Q_s^2 \sim \alpha_s^2 A^{1/3} \Lambda^2$$

LARGE SCALE

STRONG FIELD



$$= \langle A_\mu A_\mu \rangle \sim g^2 A^{1/3} \sim \alpha A^{1/3} \sim \frac{1}{\alpha} \Rightarrow A_\mu \sim \frac{1}{\sqrt{\alpha}} \sim \frac{1}{g}$$

To include multiple rescatterings in the nuclear collisions one needs to find the classical field of two ultrarelativistic nuclei.

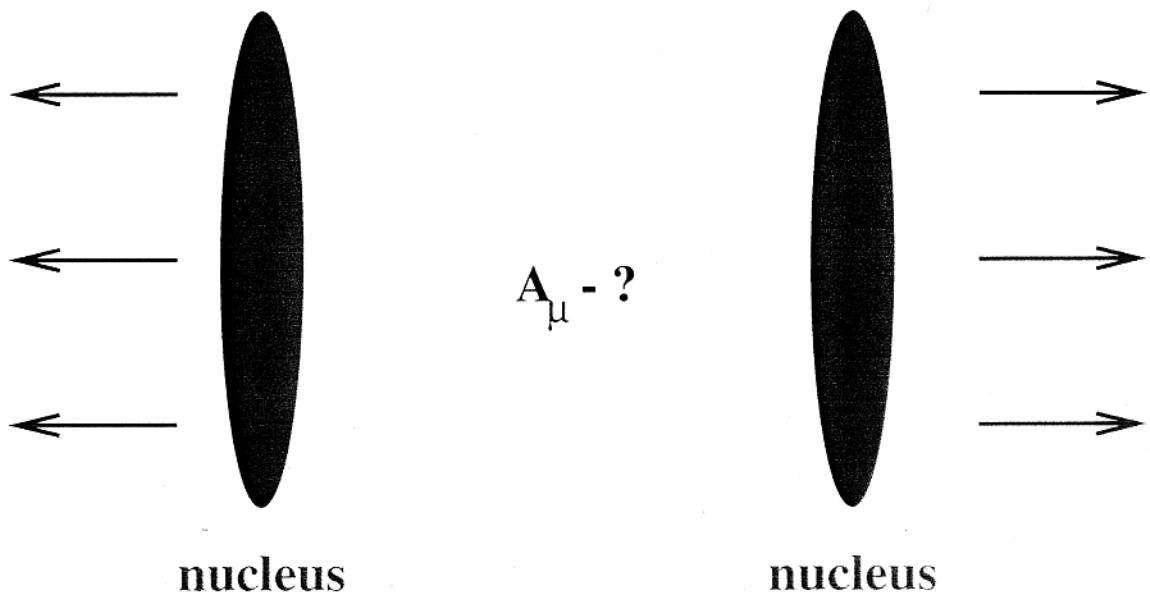


Figure 4: Classical field of two nuclei.

There have been several attempts to find the solution:

A. Kovner, L. McLerran, H. Weigert, '95; Yu. K. , D. Rischke, '97; S. Matinian, B. Muller, D. Rischke, '97; A. Krasnitz, Y. Nara, R. Venugopalan, '98-'01; S. Bass, W. Poschl, B. Muller, '98-'99; Yu. K., '00

An analytical ansatz for the multiplicity distribution of produced gluons in AA collisions has been constructed recently (Yu. K. '00).

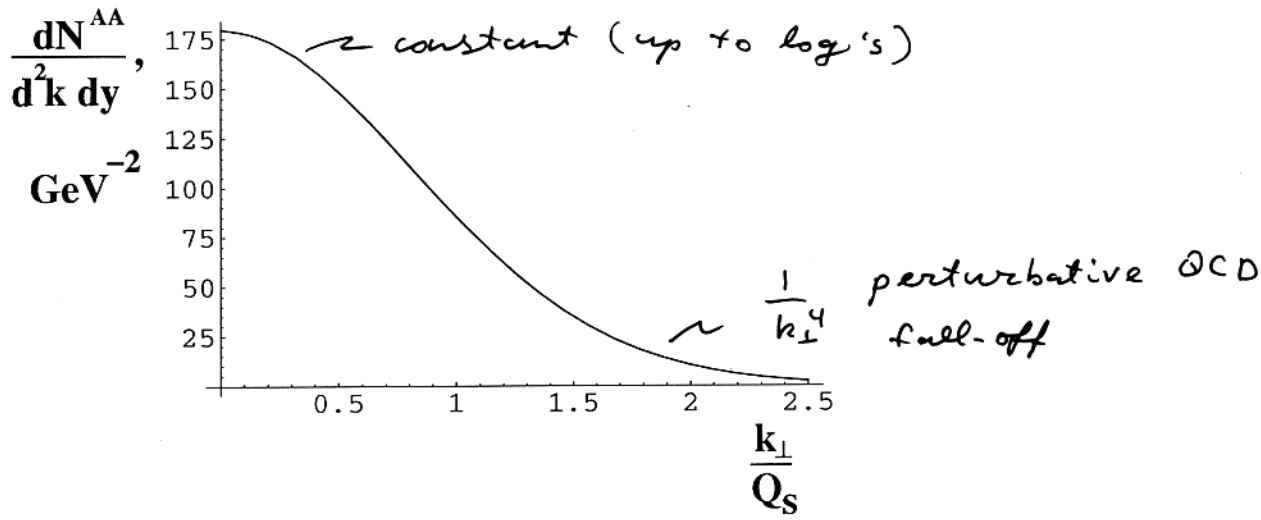


Figure 5: Distribution of the produced gluons for a central AA collision as a function of k/Q_s.

What happens to the gluons in the collision?

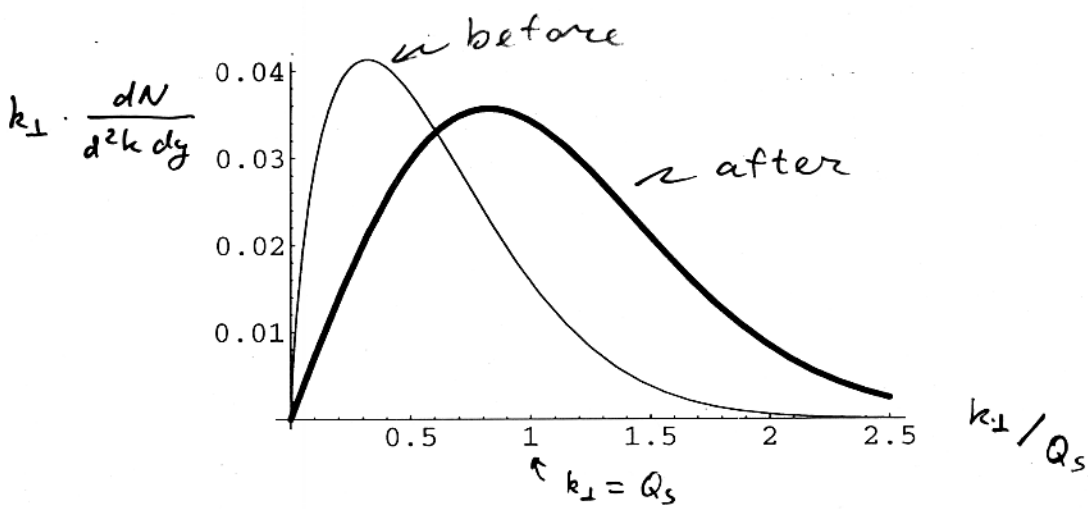
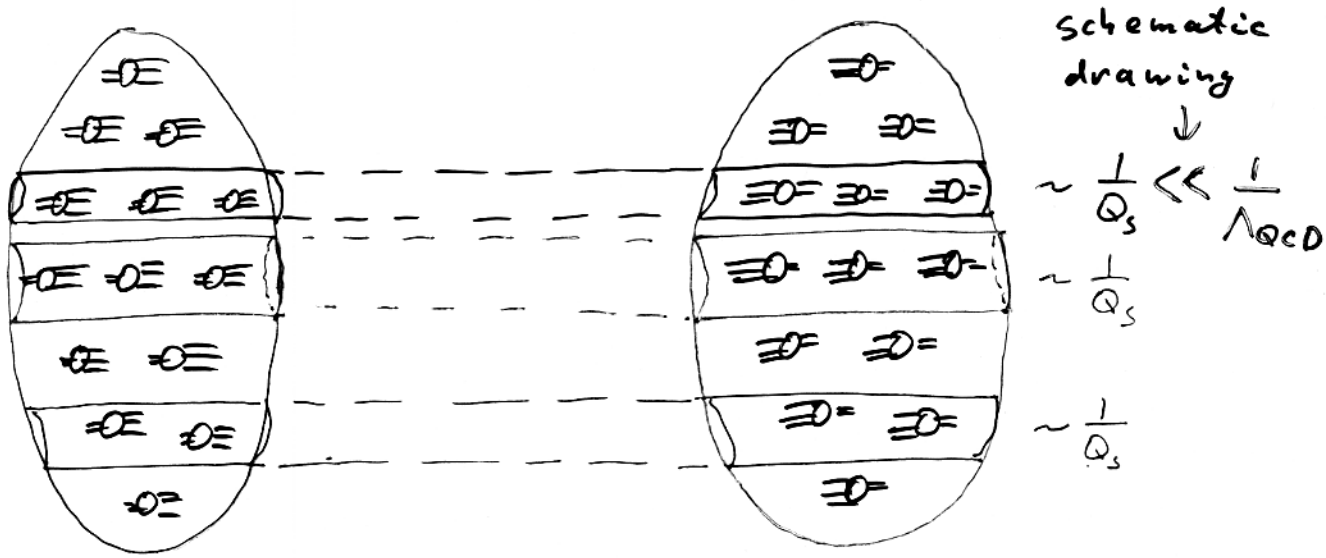


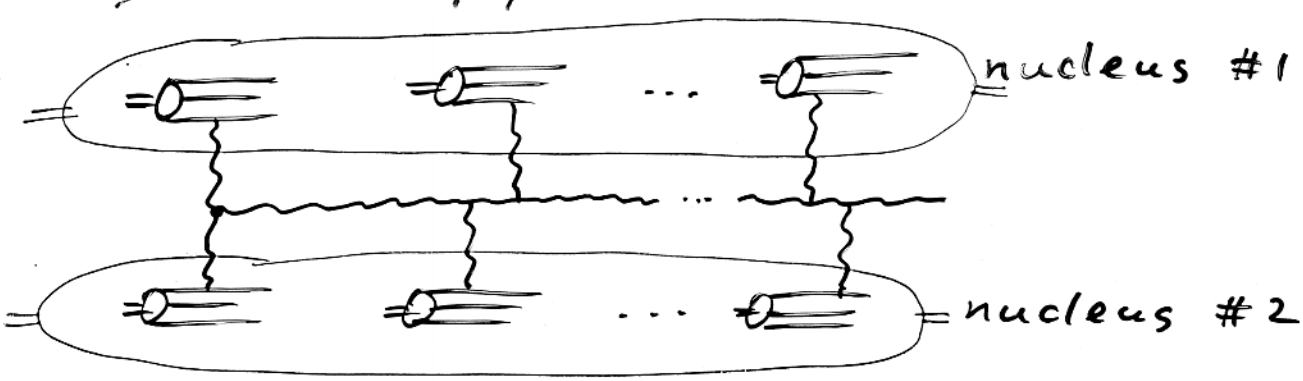
Figure 6: The gluon distribution multiplied by $k_\perp$ as a function of $k_\perp/Q_s$ in the one of the nuclei before the collision (thin line) and the gluon distribution after the collision (thick line).

What happens in AA collisions?

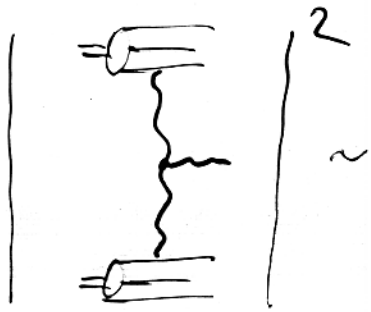


$\Rightarrow$ different "pipes" interact with each other at different $\underline{b}$ producing gluons

$\Rightarrow$ in each "pipe"



$\Rightarrow$ Estimate:



$$\sim g^6 A_1^{1/3} A_2^{1/3} \sim \alpha^3 A_1^{1/3} A_2^{1/3} \sim \frac{1}{\alpha} (\alpha^2 A_1^{1/3}) \cdot (\alpha^2 A_2^{1/3})$$

$$\Rightarrow \boxed{\frac{dN}{d^2p_T\, d^2b\, dy} \sim \frac{1}{\alpha_s}}$$

STRONG FIELDS

D. Kharzeev & E. Levin, nucl-th\0108006

Models based on saturation approach are rather successfull in describing $\frac{dN}{d\eta}\frac{1}{N_{part}}$ vs. N_{part}

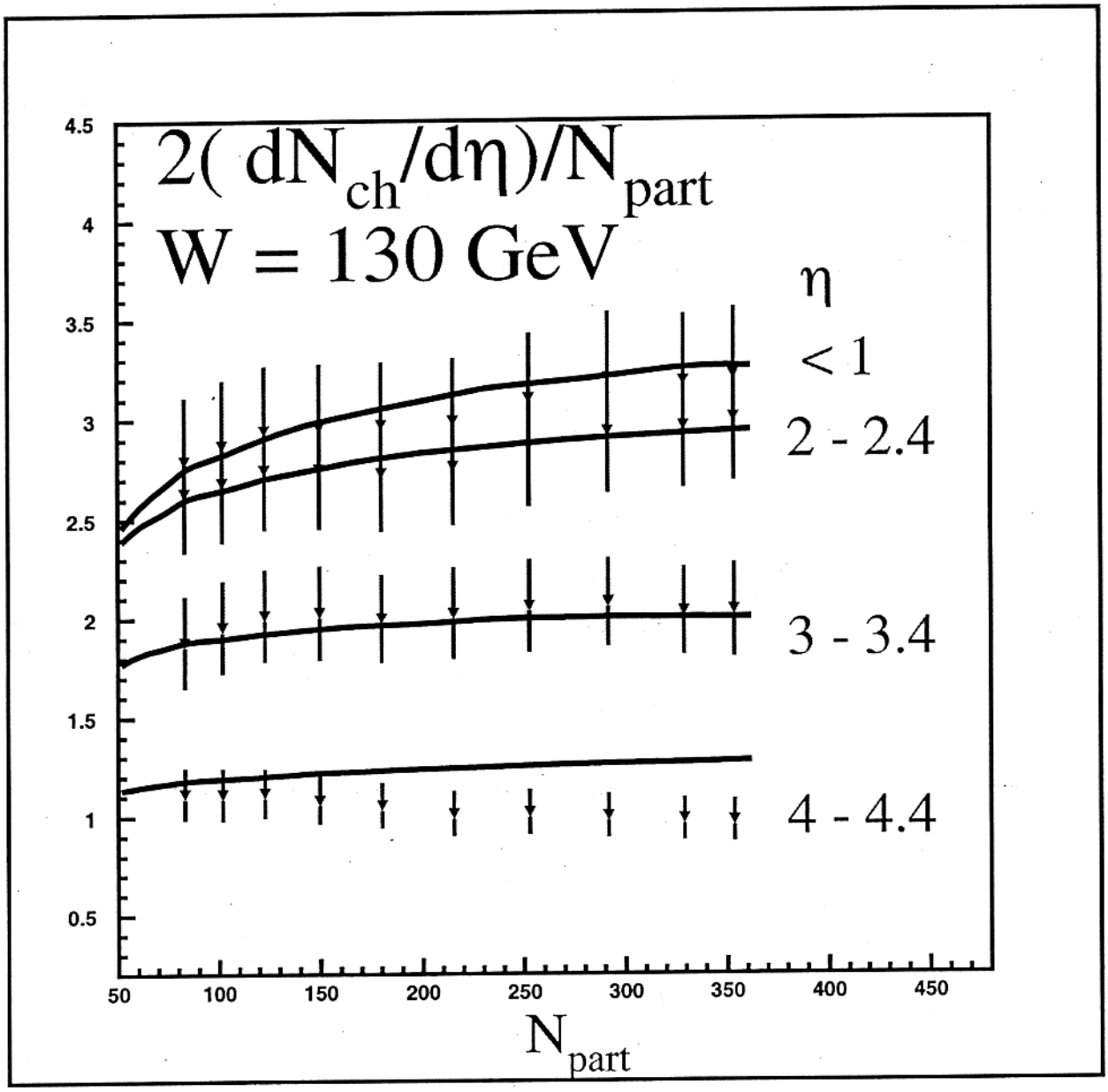


Figure 3: Centrality dependence of charged hadron production per participant at different pseudorapidity η intervals in $Au - Au$ collisions at $\sqrt{s} = 130$ GeV; the data are from [25].

and $\frac{dN}{d\eta}$ vs. η. (see also D. Kharzeev & M. Nardi '00
D. Kharzeev, E. Levin & M. Nardi '01)

Spectra ~ more involved, but may work out too...

Q_s^2 (RHIC, $\sqrt{s} = 130$ GeV) $\approx$ 1 - 2 GeV^2 borderline perturbative

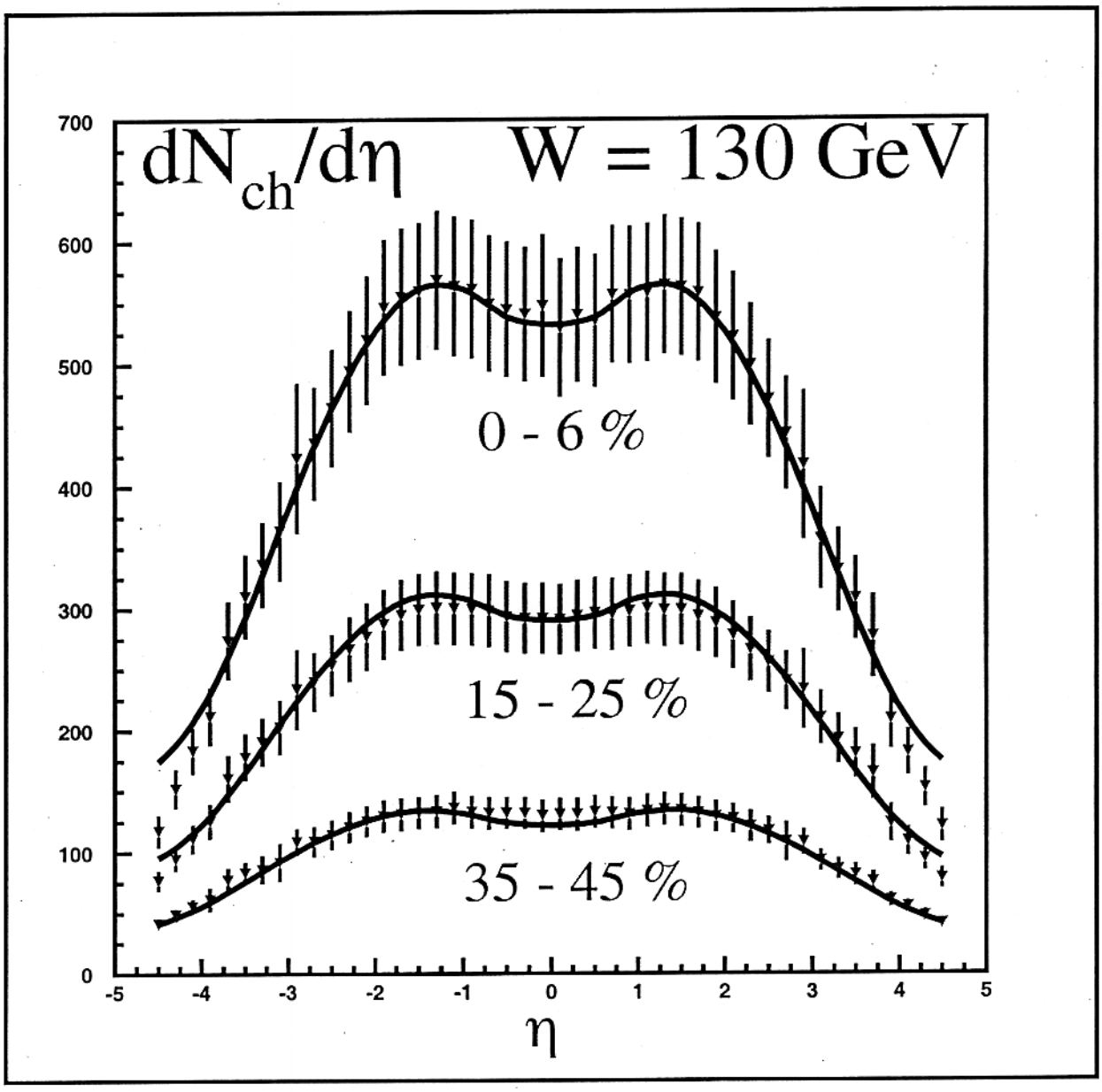


Figure 4: Pseudo–rapidity dependence of charged hadron production at different cuts on centrality in $Au - Au$ collisions at $\sqrt{s} = 130$ GeV; the data are from [25].

" Is this the whole story ? "

– M. Tannenbaum

No, this is not the whole story, since there are particle correlations! At the leading order in A classical fields are uncorrelated.

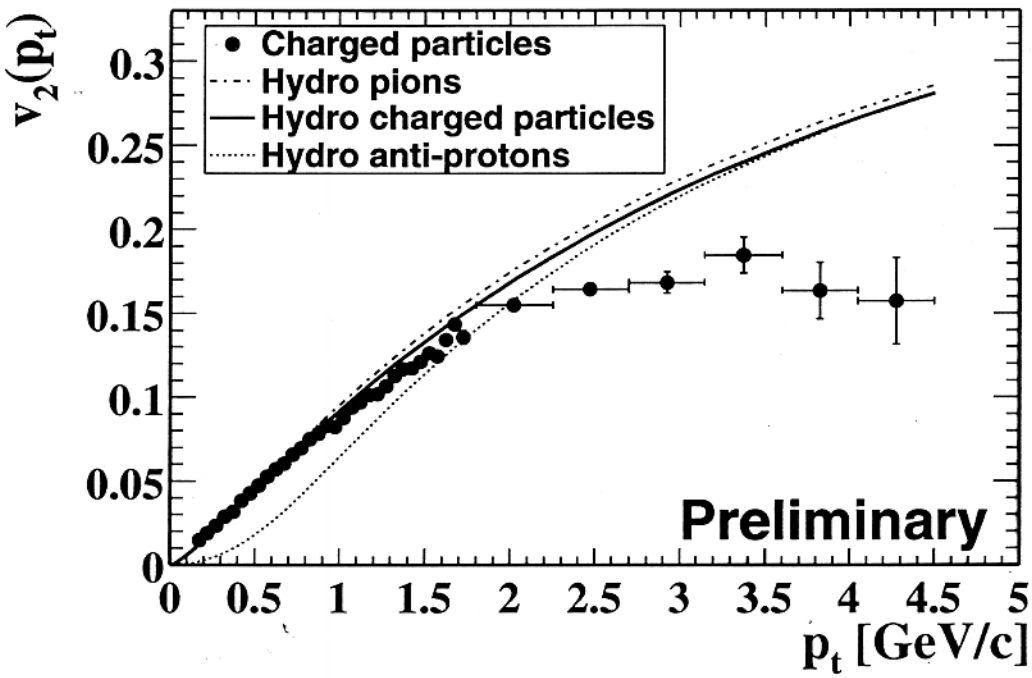


Figure 7: $v_2(p_T)$ *for charged particles and minimum bias events. R.J.M. Snellings for the STAR Collaboration, nucl-ex/0104006.*

$\Rightarrow$ Looks like a single scale distribution levelling off at $1.5 - 2$ GeV.

cf. $Q_s^2(\sqrt{s} = 130\,\text{GeV}) = 1 - 2\,\text{GeV}^2$

$\Rightarrow$ Could this be related to saturation physics?

Elliptic Flow

Definition

$$v_2(p_T) = \left\langle e^{2i(\varphi_{p_T} - \Phi_R)} \right\rangle_{events} = \frac{\int d\varphi_p\, dy \frac{dN}{d^2p_T\, dy} e^{2i(\varphi_{p_T} - \Phi_R)}}{\int d\varphi_p\, dy \frac{dN}{d^2p_T\, dy}}$$

Practical methods of determining v_2:

Traditional: $$v_2(p_T) = \frac{\langle \cos 2(\varphi_{p_T} - \Psi_R) \rangle}{\sqrt{2 \langle \cos 2(\Psi_R^a - \Psi_R^b)}}$$

where Ψ_R is reaction plane angle from whole event, Ψ_R^a & Ψ_R^b — " — from 2 different sub-event with multiplicity $\frac{N}{2}$. (N - mult. of the event)

Correlations: $$v_2(p_T) = \frac{\langle \cos 2(\varphi_1(p_T) - \varphi_2) \rangle}{\sqrt{\cos 2(\varphi_1 - \varphi_2)}}$$

S. Wang et al, '91

for pairs of particles 1 & 2, $\varphi_1(p_T)$ means fixed p_T

⇝ Methods are identical iff either

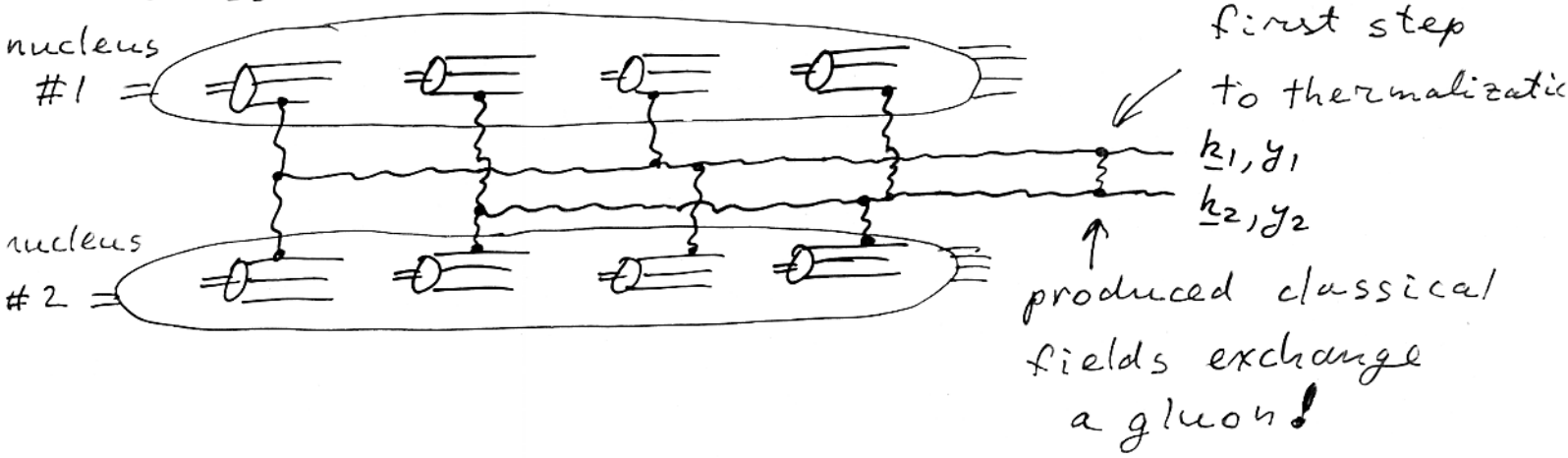
$$v_2 \ll \frac{1}{\sqrt{N}} \quad \text{(weak flow)}$$

or $$v_2 \sim \frac{1}{\sqrt{N}} \quad \text{(non-flow correlations)}$$

⇝ they are identical at RHIC! (see STAR nucl-ex/0206001

⇝ We'll stick to the 2nd method/definition.

The most basic correlations between classical fields:



$$\frac{dN}{d^2k_1 dy_1\, d^2k_2 dy_2} = \underbrace{\frac{dN}{d^2k_1 dy_1}\,\frac{dN}{d^2k_2 dy_2}}_{\text{leading in A piece}} + \underbrace{\frac{dN_{corr}}{d^2k_1 dy_1\, d^2k_2 dy_2}}_{\text{correlations pictured above}}$$

leading in A piece ~ uncorrelated classical fields, NO INTERACTION

Let us do some estimates:

remember $\frac{dN}{d^2k\, dy\, d^2b} \sim \frac{1}{\alpha_s} \Rightarrow \frac{dN}{d^2k\, dy} \sim \frac{S_\perp}{\alpha_s} \sim \frac{A^{2/3}}{\alpha_s} \sim \frac{1}{\alpha_s^5}$ (overlap area)

as $\alpha_s^2 A^{1/3} \sim 1$.

$\Longrightarrow$ Leading piece $\sim \left(\frac{dN}{d^2k\, dy}\right)^2 \sim \frac{1}{\alpha_s^{10}}$

$\Longrightarrow$ Our non-flow correlations

$$\frac{dN_{corr}}{d^2k_1 dy_1\, d^2k_2 dy_2} \sim \int d^2b\, \underbrace{\frac{dN}{d^2k_1 d^2b\, dy_1}}_{\sim 1/\alpha_s}\, \underbrace{\frac{dN}{d^2k_2 d^2b\, dy_2}}_{\sim 1/\alpha_s} \cdot \alpha_s^2 \sim S_\perp \sim \frac{1}{\alpha_s^4}$$

(gluon exchange)

only one $\underline{b}$ ~ gluon is local

$$\frac{dN_{corr}}{d^2k_1 dy_1\, d^2k_2 dy_2} \sim \frac{1}{\alpha_s^4}$$

$$\left(\frac{dN}{d^2k\,dy}\right)^2 \sim \frac{S_\perp^2}{\alpha_s^2} \sim \frac{1}{\alpha_s^{10}} \quad \text{and} \quad \frac{dN_{corr}}{d^2k_1\,dy_1\,d^2k_2\,dy_2} \sim S_\perp \sim \frac{1}{\alpha_s^4}$$

Contribution to flow:

$$\boxed{v_2^{int} \sim \sqrt{\frac{\frac{dN_{corr}}{d^3k_1\,d^3k_2}}{\frac{dN}{d^3k_1}\,\frac{dN}{d^3k_2}}} \sim \frac{\alpha_s}{\sqrt{S_\perp}} \sim \alpha_s^3}$$

← our flow estimate

Is there other correlations?

⇝ Yes. Classical field knows about geometry!

"typical" modes $p_T \sim Q_s \Rightarrow \Delta x_\perp \sim \frac{1}{p_T} \sim \frac{1}{Q_s} \ll \frac{1}{\Lambda_{QCD}} \ll R$

(don't know about geometry)

ultra-soft modes $p_T \sim \frac{1}{R} \Rightarrow \Delta x_\perp \sim R$

(know about geometry!)

but: ⇝ $p_T \sim \frac{1}{R}$ ~ are those still gluonic? maybe ...

$\alpha_s = \alpha_s(Q_s)$

$$\rightsquigarrow \quad \frac{dN}{d^2p_T\,dy} \sim \left.\frac{dN}{d^2p_T\,dy}\right|_{R\to\infty} \left(1 + o\left(\frac{1}{R^2}\right)\right)$$

leading order $p_T \sim Q_s$ $\sim \frac{S_\perp}{\alpha} \sim \frac{1}{\alpha^5}$

subleading shape-dependent correction $\sim \frac{1}{R^2}\cdot\frac{S_\perp}{\alpha} \sim \frac{1}{\alpha}$

Contribution to flow

$$\boxed{v_2^{shape} \sim \frac{o(1/R^2)\,\frac{dN}{d^2p_T\,dy}}{\frac{dN}{d^2p_T\,dy}} \sim \frac{1}{R^2} \sim \alpha_s^4}$$

Teaney & Venugopala[illegible] '02

$$\frac{v_2^{shape}}{v_2^{int}} \sim \alpha_s \ll 1 \Rightarrow v_2^{shape} \ll v_2^{int}$$

could still be important as $\alpha_s \approx .3$

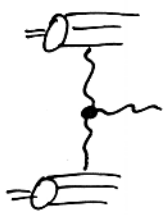
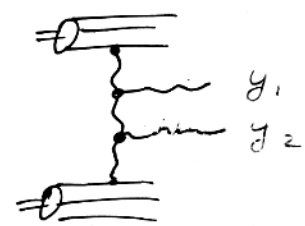
[illegible] relevant at high p_T, as $p_T \sim \frac{1}{R}$

Our model

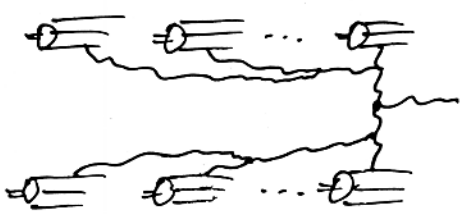
⇝ Unfortunately nobody knows how to correctly calculate 2-gluon inclusive production in AA collisions including all multiple rescatterings..

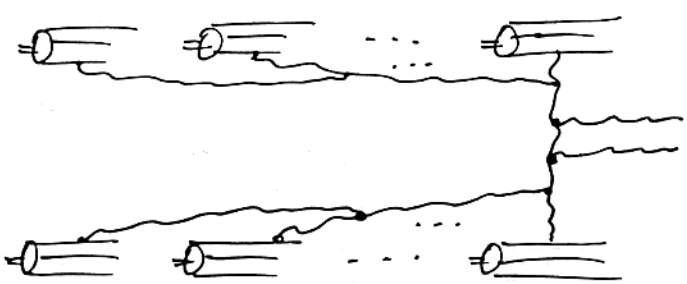
⇝ We have to construct a realistic model
(cf. Kharzeev & Levin, '01)

Take lowest order graphs $(y_1 \gg y_2)$

$\frac{dN}{d^2k\,dy} \sim$  , $\frac{dN_{corr}}{d^2k_1 dy_1 d^2k_2 dy_2} \sim$ 

and "dress" them with classical field gluon distributions ("form factors")

$\frac{dN}{d^2k\,dy} \sim$  1 particle inclusive

$\frac{dN_{corr}}{d^2k_1\,dy_1\,d^2k_2\,dy_2} \sim$  2 particles inclusive

⇝ Crude approximation, but works phenomenological

above saturation scale, $p_T \gg Q_s$

$$\frac{dN}{d^2k\,dy} \sim S_\perp \frac{Q_s^4}{k_\perp^4} \quad \text{and} \quad \frac{dN_{corr}}{d^2k_1 dy_1 d^2k_2 dy_2} \sim S_\perp \frac{Q_s^4}{\underline{k}_1^2 \underline{k}_2^2 (\underline{k}_1+\underline{k}_2)^2}$$

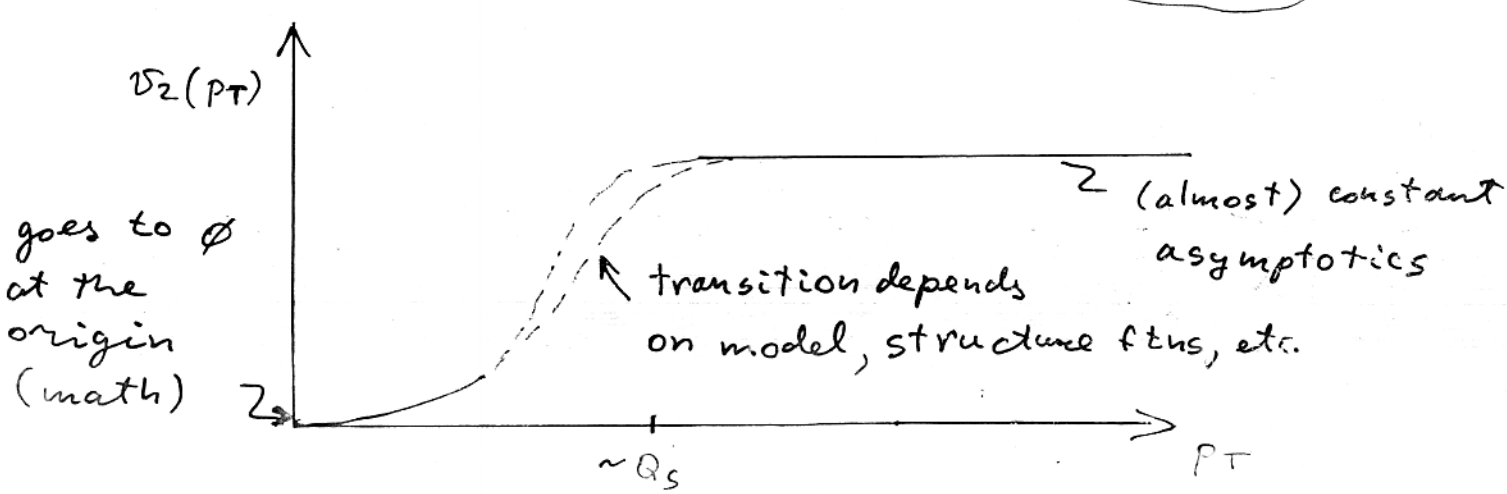
$$v_2(k_1) \propto \left[\frac{\int d^2k_2\, dy_2\, dy_1 \frac{dN_{corr}}{d^2k_1 dy_1 d^2k_2 dy_2} \cos 2(\varphi_1-\varphi_2)}{\int \frac{dN}{d^2k_1 dy_1} dy_1 \int d^2k_2\, dy_2 \frac{dN}{d^2k_2 dy_2}} \right]^{1/2}$$

$$\sim \left[\frac{\int d^2k_2 \frac{1}{\underline{k}_1^2 \underline{k}_2^2 (\underline{k}_1+\underline{k}_2)^2} \cos 2(\varphi_1-\varphi_2)}{S_\perp Q_s^4 \frac{1}{\underline{k}_1^4} \underbrace{\int_{Q_s} d^2k_2 \frac{1}{k_2^4}}_{k_1\text{-independent} \sim \frac{1}{Q_s^2}}} \right]^{1/2}$$

$$\sim \frac{1}{\sqrt{S_\perp Q_s^2}} \left(\frac{1/k^4}{1/k_1^4} \right)^{1/2} \sim \frac{\text{const}(k_1)}{\sqrt{S_\perp Q_s^2} \sim N_{part}} \quad \text{up to log's}$$

$\Longrightarrow$ for $p_T \gg Q_s$ $\quad v_2(p_T) \sim \text{const}$

$\Longrightarrow$ for $p_T \ll Q_s$ $\quad v_2(p_T) \sim p_T^2 \ln \frac{Q_s}{p_T}$



also in agreement with PHENIX

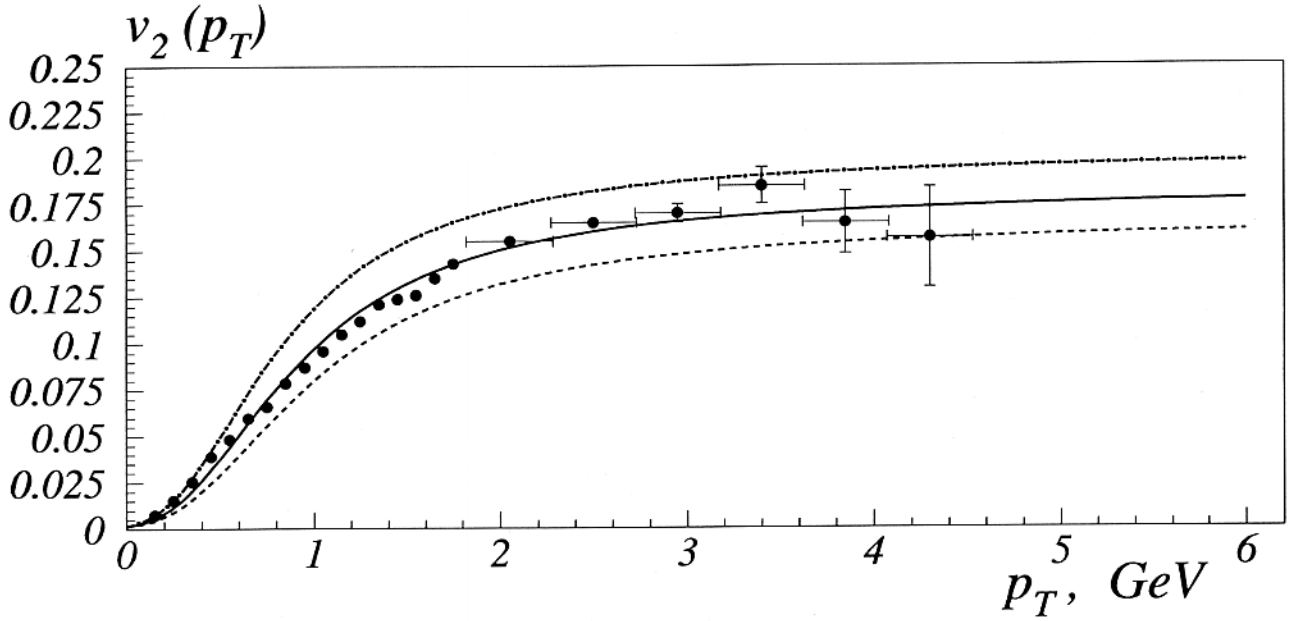


Figure 8: *Differential elliptic flow data from STAR versus the predictions of our minijet model. Different lines correspond to our predictions with different values of the saturation scale:* $Q_s = 1.0$ *GeV (solid line),* $Q_s = 1.1$ *GeV (dashed line) and* $Q_s = 0.9$ *GeV (dash-dotted line). We used the following parameters* Λ*=0.15 GeV,* α_S*=0.3,* A*=197 (Gold).*

$$v_2(p_T) = \alpha_S \left(\frac{\pi^2 N_c K_2}{2 \ln 2 \, C_F \, S_\perp^A \, Q_s^2 \, K_1^2} \right)^{1/2}$$

$$\frac{\int_0^\infty \frac{dz}{z^3} J_2(p_T z) \left(1 - e^{-\underline{z}^2 Q_s^2/4}\right)^2}{\int_0^\infty \frac{dz}{z^3} J_0(p_T z) \left(1 - e^{-\underline{z}^2 Q_s^2/4}\right)^2}$$

very peripheral
~ non-perturbative

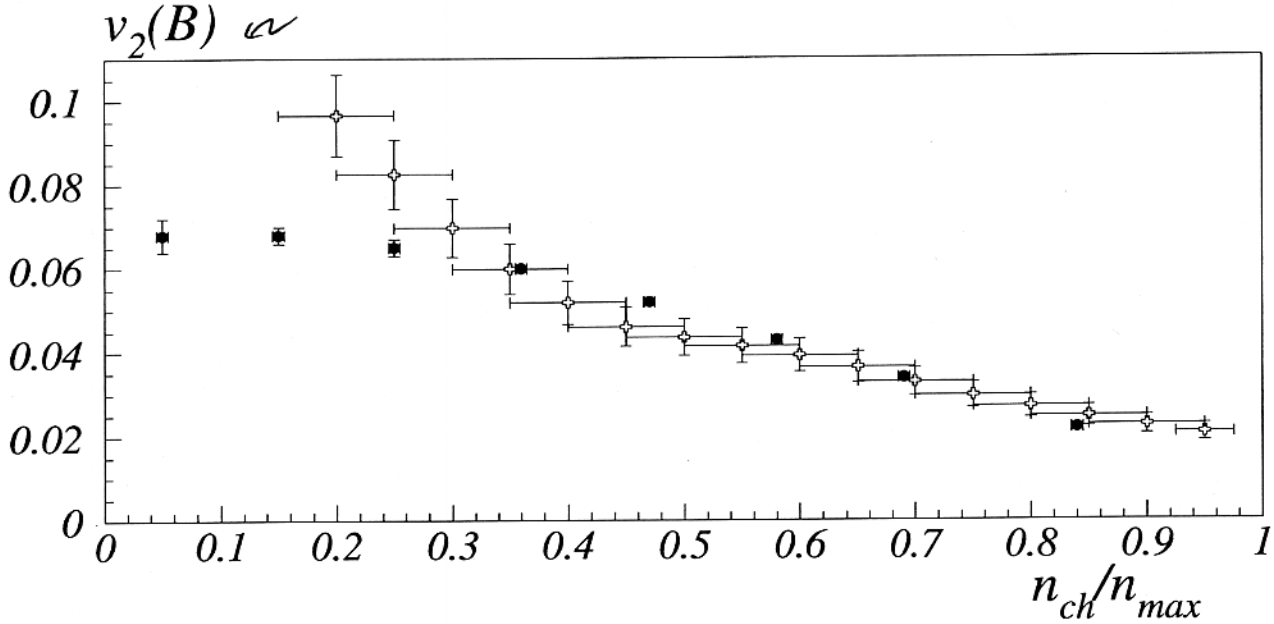


Figure 9: *Centrality dependence of elliptic flow given by the STAR data (black dots) and our fit (empty crosses). We used Λ=0.15 GeV, α_S=0.3, A=197, Q_s=1 GeV and $N_{\rm part}(B=0)$ = 344.*

For a cylindrical nucleus we get $\left(v_2 \sim \frac{1}{\sqrt{S_\perp Q_s^2}} \right)$

$$v_2(B) \sim \frac{1}{\sqrt{N_{part}}}$$

which gives

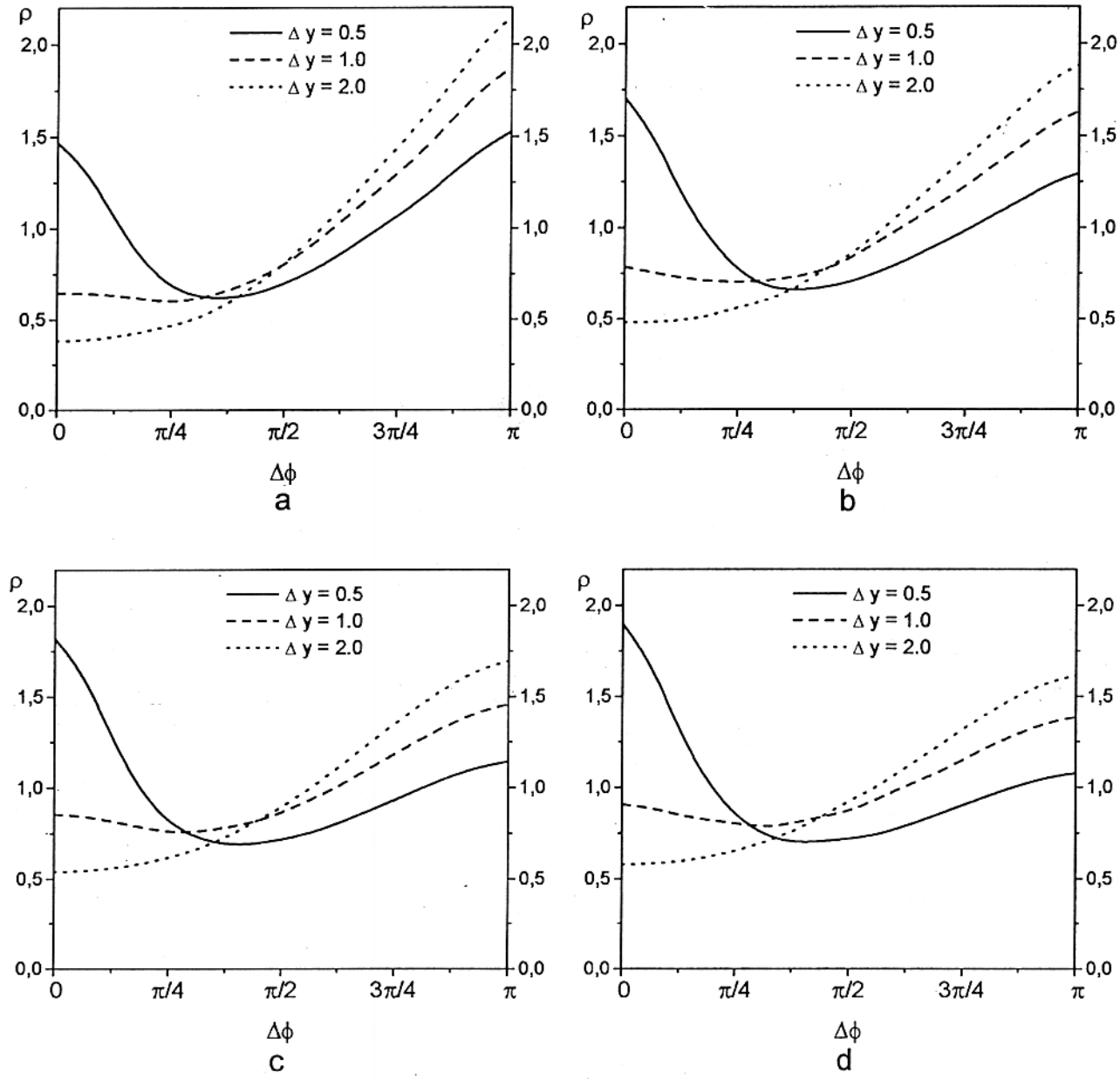
$$v_2(B) = \alpha_S \left(\frac{\pi\, N_c\, K_2\, N_{\rm part}(B=0)}{2 \ln 2\, C_F\, S_\perp^A\, Q_s^2\, N_{\rm part}} \right)^{1/2}$$

as in PHENIX and STAR analyses

$$\rho(\Delta\varphi) \sim \frac{d\sigma}{d\Delta\varphi\, dy_1 dy_2} \sim C(\Delta\varphi)$$

hep-ph/9905496

from Leonidov & Ostrovsky, '99



for pp collisions

Figure 2

⇒ a more realistic calculation with $y_1 \sim y_2$ ($\eta_1 \sim \eta_2$) gives better-looking $C(\Delta\varphi)$!

Higher order cumulants

⇝ Borghini, Dinh, Ollitrault '99-'02

$$c_4 = \langle\langle e^{2i(\varphi_1+\varphi_2-\varphi_3-\varphi_4)} \rangle\rangle \overset{def}{\equiv} \langle e^{2i(\varphi_1+\varphi_2-\varphi_3-\varphi_4)} \rangle -$$

$$- 2 \langle e^{2i(\varphi_1-\varphi_3)} \rangle \langle e^{2i(\varphi_2-\varphi_4)} \rangle = -(v_2)^4$$

(flow only)

⟹ if flow is large, much larger than non-flow correlations, than non-flow effects should be suppressed in high-order cumulants:

$$v_2^{\text{non-flow}}(\text{from } c_2) \sim \frac{1}{\sqrt{N}}, \quad v_2^{\text{non-flow}}(\text{from } c_4) \sim \frac{1}{N^{3/4}}, \ldots$$

⟹ however, if flow is small and non-flow correlations are large the method does not work!

⇝ <u>model</u> with only 2-particle non-flow correlations

$$\frac{dN}{d^3k_1\, d^3k_2} = \frac{dN}{d^3k_1}\frac{dN}{d^3k_2} + \frac{dN_{corr}}{d^3k_1\, d^3k_2} + \text{nothing (stop here)}$$

$$c_4 = \frac{2\int d^3k_1 \ldots d^3k_4 \, \frac{dN_{corr}}{d^3k_1\, d^3k_3}\, \frac{dN_{corr}}{d^3k_2\, d^3k_4}\, e^{2i(\varphi_1+\varphi_2-\varphi_3-\varphi_4)}}{\int \frac{dN}{d^3k_1}\frac{dN}{d^3k_2}\frac{dN}{d^3k_3}\frac{dN}{d^3k_4} + \binom{4}{2} = 6 \int \frac{dN}{d^3k_1}\frac{dN}{d^3k_2}\frac{dN_{corr}}{d^3k_3\, d^3k_4} + \text{h.o.}} -$$

$$- \frac{2\int \frac{dN_{corr}}{d^3k_1\, d^3k_3}\frac{dN_{corr}}{d^3k_2\, d^3k_4}\, e^{2i(\varphi_1+\varphi_2-\varphi_3-\varphi_4)}}{\left[\int \frac{dN}{d^3k_1}\frac{dN}{d^3k_2} + \frac{dN_{corr}}{d^3k_1\, d^3k_2}\right]^2} \approx -8 \frac{\int \left[\frac{dN_{corr}}{d^3k_1\, d^3k_2}\, e^{2i(\varphi_1-\varphi_2)}\right]^2 \frac{dN}{d^3k_1\, d^3k_2}}{\int \left(\frac{dN}{d^3k}\right)^2} = -v_2^4$$

when dust settles

$$V_2\,(\text{from } C_4) = V_2\,(\text{from } C_2)\cdot\left[\frac{8\int \frac{dN_{corr}}{d^3k_1\, d^3k_2}}{\int \frac{dN}{d^3k_1}\,\frac{dN}{d^3k_2}}\right]^{1/4}$$

$\Rightarrow$ in our model $\dfrac{\int \frac{dN_{corr}}{d^3k_1\, d^3k_2}}{\int \frac{dN}{d^3k_1}\,\frac{dN}{d^3k_2}} \approx \dfrac{1}{50}$ (central collisions)

$$\Rightarrow \quad V_2\,(\text{from } C_4) \approx 0.63 \cdot V_2\,(\text{from } C_2)$$

NO 4-particle correlations at all

$\rightsquigarrow$ still get $V_2\,(\text{from } C_4) \approx 63\%\ V_2\,(\text{from } C_2)$

$\Rightarrow$ more conservative: at RHIC central

$V_2 \approx 3\%$ $\Rightarrow$ $\dfrac{\int \frac{dN_{corr}}{d^3k_1\, d^3k_2}}{\int \frac{dN}{d^3k_1}\,\frac{dN}{d^3k_2}} \gtrsim V_2^2 \approx .0\overset{00}{\underset{\ell}{9}}$

get $V_2\,(\text{from } C_4) \approx 0.29\ V_2\,(\text{from } C_2)$

$\rightsquigarrow$ numbers are consistent with

STAR higher-cumulant analysis

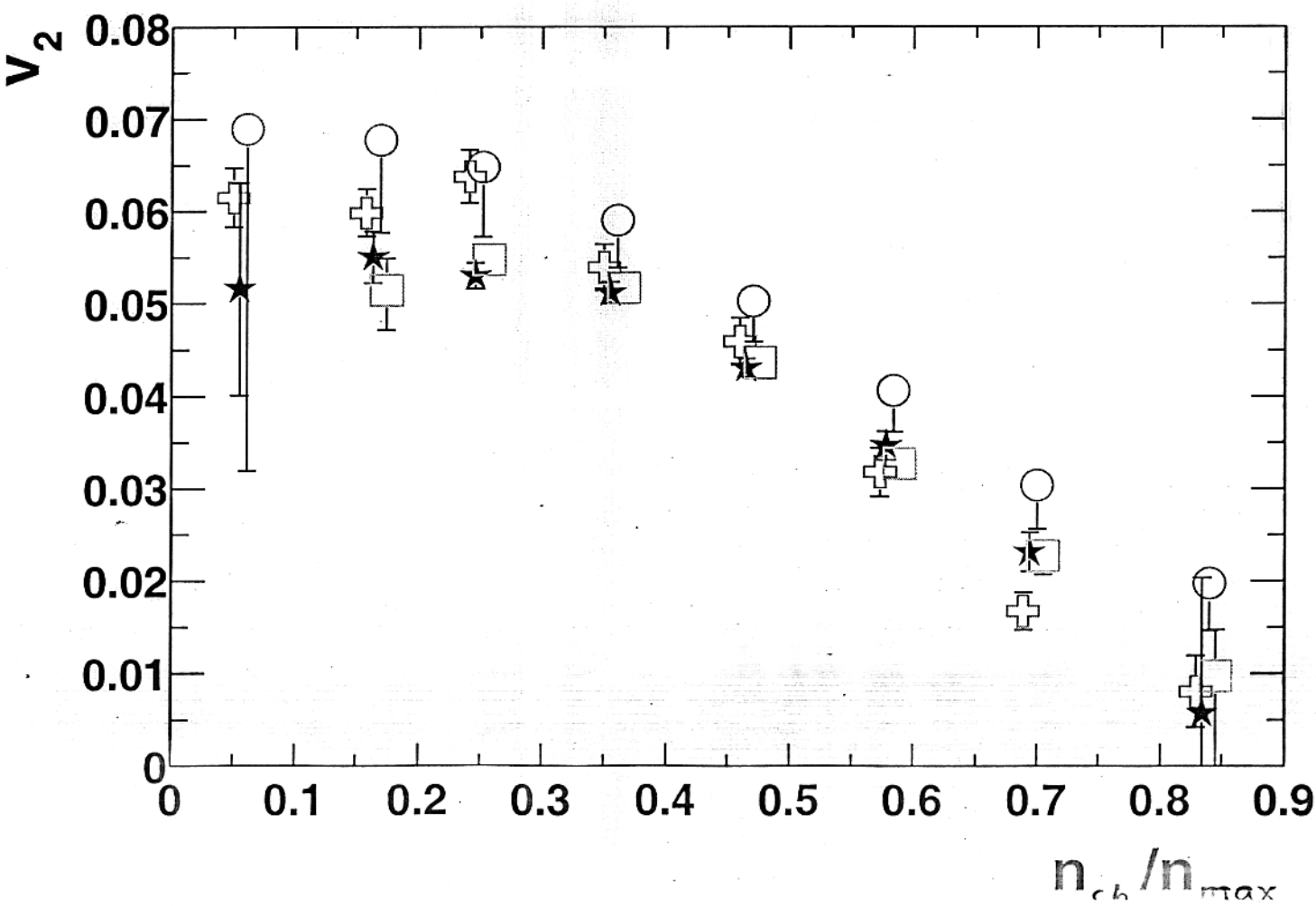
STAR, nucl-ex/0206001

○ ~ traditional flow analysis

★, □ ~ 4th order cumulants

most central $\frac{v_2(c_4)}{v_2(c_2)} \sim 25-50\%$

in agreement with our toy model



C O N C L U S I O N S

$\Rightarrow$ constructed a simple model with basic final state interactions & local correlations

$\Rightarrow$ was able to describe the data for $v_2(p_T)$ with turnover of v_2 at Q_s and flattenning at $p_T > Q_s$

$\Rightarrow$ centrality dependence of $v_2(B)$ is in reasonable agreement with the data