Hadronic Transport Model with a Phase Transition

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- General Idea
- Thermodynamic Properties
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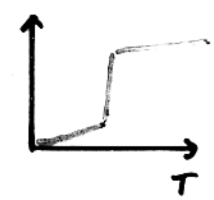
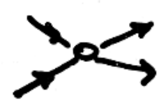
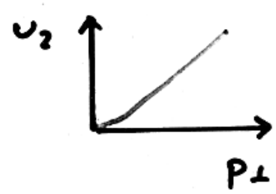






General Idea

Info available on: STRONGLY INT MATTER

- Hadronic Interactions at Low Densities
  Transport theory with mean fields and binary interactions has had many successes in describing data
- Primarily Baryonless Matter around and above T_c from Lattice Calculations
- Ground-State Nuclear Matter

Info lacking on:

- Matter at high ρ out of Equilibrium
- Hadronization Process
- Large μ and T Matter

FREEZE-OUT

Idea: Hadronic observables probe directly low-ρ stage of the reaction and are sensitive to high-ρ stage only on average.

$\Rightarrow$ Hadronic model roughly exact in low-ρ limit and thermodynamically consistent with the other explored limits.

Hadronic mean fields (MFs) may be used to vary the thermodynamic properties at high ρ and cross sections to vary transport properties.

Hadron-to-QG PT characterized by rapid rise in No. of degrees of freedom (DOF) and reduction in m of DOF.

We use MFs to lower hadron m's.
More hadrons are then produced at a given T.

To match approx DOF No. in QGP,
24 q's + 16 g's = 40, we apply cut-off in hadron spectrum and include N, $\overline{\text{N}}$, Δ, $\overline{\Delta}$, π, and ρ.

When m low, DOF No.: $8 + 32 + 3 + 9 = 52$

Thermodynamic Properties

Relativistic Landau Theory

$$T^{00} = e \equiv e\{f\}$$

$$\epsilon^i_{\mathbf{p}} = \frac{\delta e}{\delta f^i(\mathbf{p}, \mathbf{r}, t)}$$

SINGLE-PTCLE ENERGY

e – volume energy density, f^i – phase-space density, $(\mathbf{p}, \epsilon_{\mathbf{p}})$ – 4-vector

Simple parametrization of the energy density in the local rest-frame:

$$e = \sum_i \int d\mathbf{p}\, \epsilon^i_{\mathbf{p}}\, f^i(\mathbf{p}) + e_s(\rho_s) + e_v(\rho_v)$$

where

SCALAR

$$\rho_s = \sum_i \int d\mathbf{p}\, \frac{m^i\, m^i_0}{\sqrt{m^{i2} + p^2}}\, f^i(\mathbf{p})$$

VECTOR ≡ BARYON DENSITY

$$\rho_v = \sum_i B^i \int d\mathbf{p}\, f^i(\mathbf{p})$$

Two densities, ρ_s and ρ_v, needed to be parametrize independently the thermodynamic properties along the $\mu = 0$ and $T = 0$ axes. Two model versions: with and without lowering of π masses.

With the adopted parametrization:

$$m^i = m_0^i \, S(\rho_s) \, , \quad \text{where} \quad S = \int \frac{d\rho_s}{\rho_s} \frac{de_s}{d\rho_s}$$

and, in the local rest-frame,

$$\epsilon_{\mathbf{p}}^i = \sqrt{m^{i2} + p^2} + B^i \, V(\rho_v) \, , \text{ where } V = \int \frac{d\rho_v}{\rho_v} \frac{de_v}{d\rho_v}$$

In any frame

$$\rho_v^\nu = \left(\sum_i \int d\mathbf{p} \, f^i , \sum_i \int d\mathbf{p} \, \frac{\delta \epsilon^i}{\delta \mathbf{p}} \, f^i \right) , \quad \rho_v^\nu \, \rho_{v\nu} = \rho_v^2$$

CANONICAL KINEMATIC

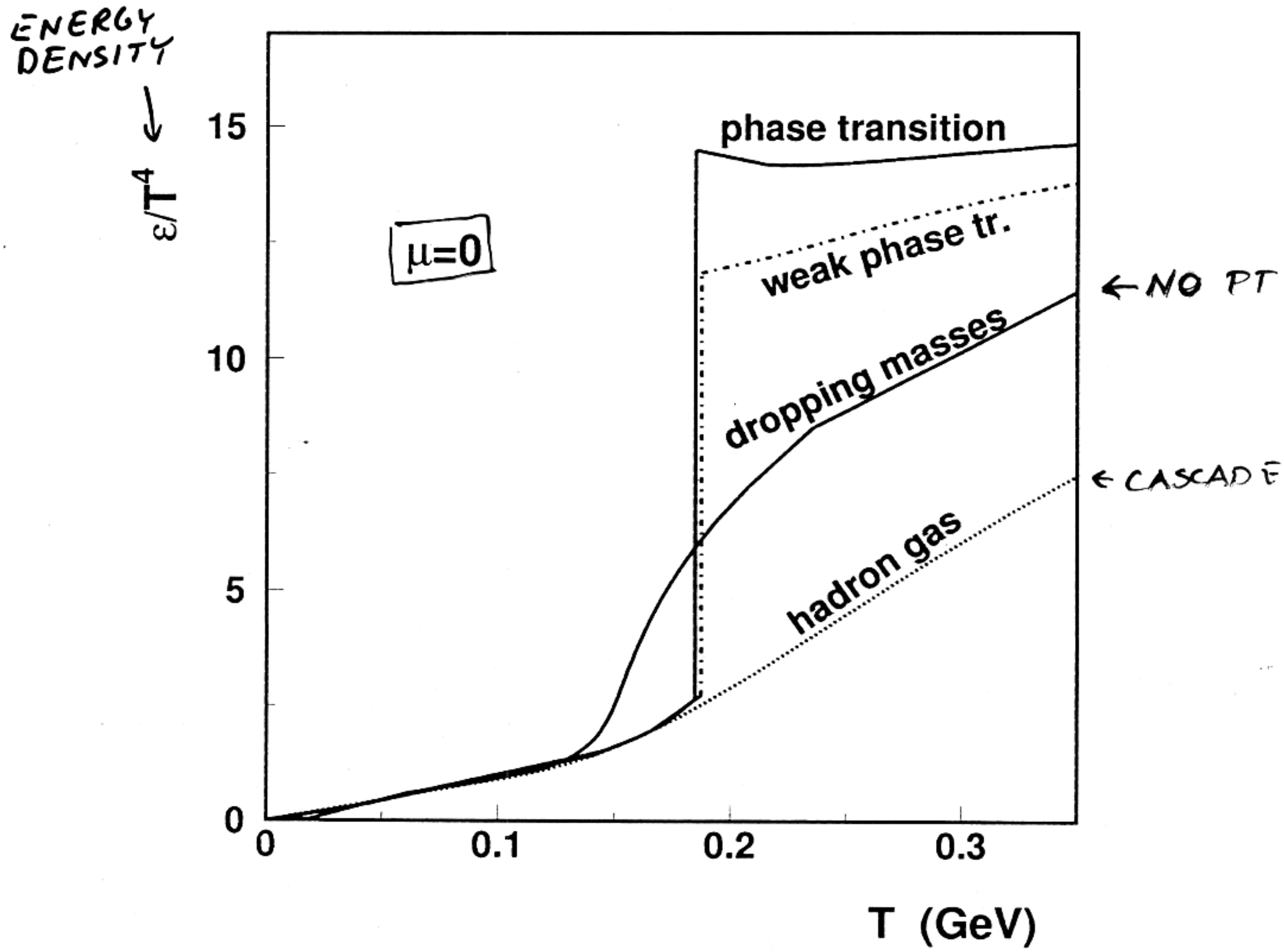
$$p^{i\nu} = p^{i*\nu} + B^i \, V \, \rho_v^\nu / \rho_v \, , \quad \text{with} \quad p^{i*2} = m^{i2}$$

Locally: $\mathbf{p}^{i*} = \mathbf{p}$.

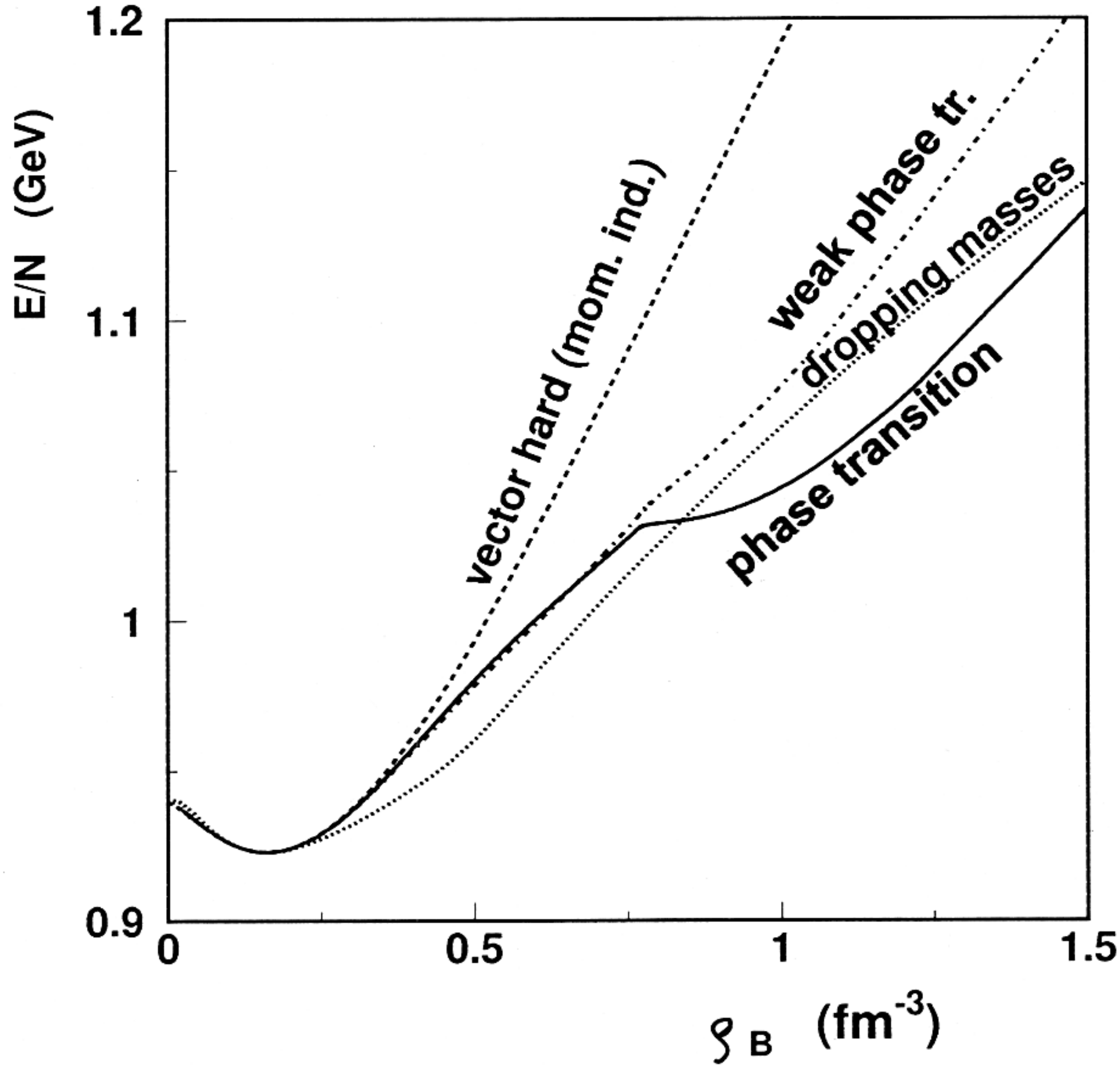
With regard to thermodynamic properties: a generalization of the Walecka model.

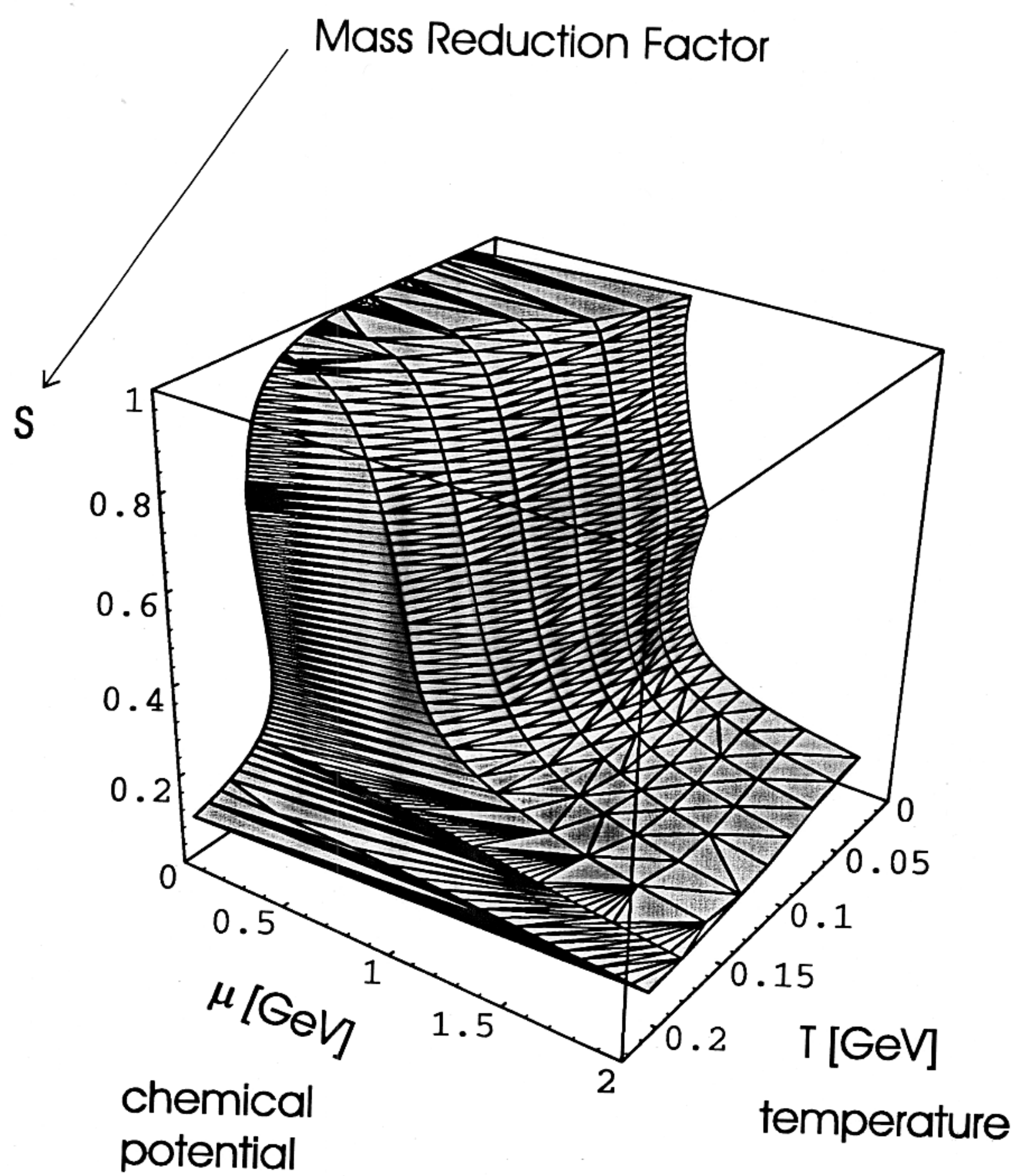
← MASS REDUCTION FACTOR

Different EOS from different $S(\rho_s)$:

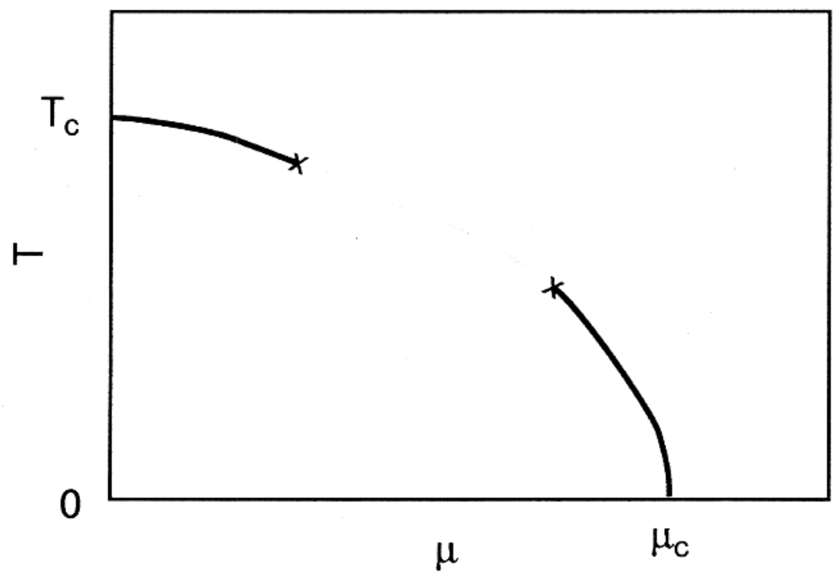


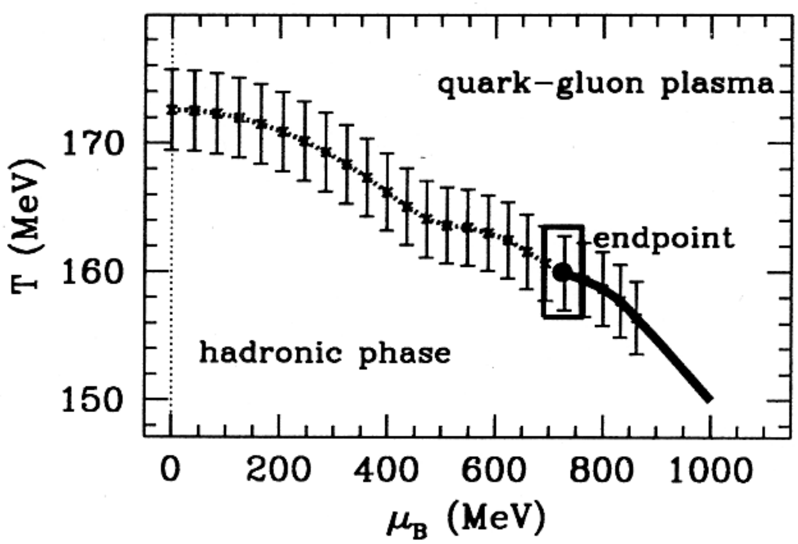
Different $T = 0$ EOS for different S and V:




Mass Reduction Factor
S
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
μ [GeV]
chemical
potential
0
0.05
0.1
0.15
0.2
T [GeV]
temperature

Our model: 2 tricritical pts





Fodor & Katz, lattice

Transport Theory

Boltzmann Eq.: same general form relativistically as nonrelativistically:

COLL RATE

$$\frac{\partial f}{\partial t} + \underbrace{\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}}}_{\text{VELOCITY}} \frac{\partial f}{\partial \mathbf{r}} - \underbrace{\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}}}_{\text{FORCE}} \frac{\partial f}{\partial \mathbf{p}} = I$$

IN LOCAL BARYON FRAME
$\vec{p}^{\,*} = \vec{p}$
$\epsilon_p^* + V_0 = \epsilon_p$

In terms of kinematic vbles:

KIN ENERGY

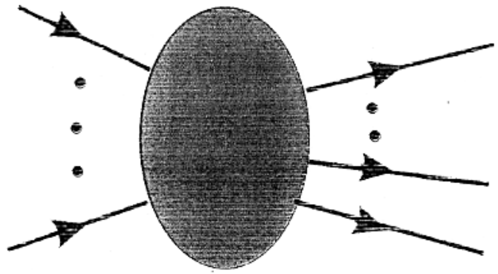
$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}^*}{\epsilon^*_{\mathbf{p}}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial}{\partial \mathbf{r}} \left(\epsilon^*_{\mathbf{p}} + V^0\right) \frac{\partial f}{\partial \mathbf{p}^*} = I$$

POTENTIAL

I – collision rate. All functions refer to one location $(\mathbf{r}, t)$ in space-time.

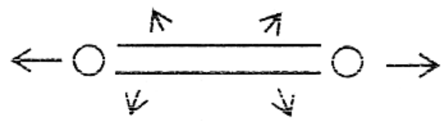
Walecka m. before: Ko, Li, Wang, PRL59, 1084 (87)

Collision Processes



$$
\begin{aligned}
I \;=\; & \sum_{n,n'\geq 2} \int \frac{d\mathbf{p}_2}{\gamma_2} \dots \frac{d\mathbf{p}_n}{\gamma_n} \int \frac{d\mathbf{p}'_1}{\gamma'_1} \dots \frac{d\mathbf{p}'_{n'}}{\gamma'_{n'}} |\mathcal{M}|^2 \\
& \times \delta\left(\sum_{i'=1}^{n'} p'_{i'} - \sum_{i=1}^{n} p_i \right) (f'_1 \dots f'_{n'} - f_1 \dots f_n) \\
= \; & \sum_{n,n'\geq 2} \int \frac{d\mathbf{p}^*_2}{\gamma_2} \dots \frac{d\mathbf{p}^*_n}{\gamma_n} \int \frac{d\mathbf{p}^{*'}_1}{\gamma'_1} \dots \frac{d\mathbf{p}^{*'}_{n'}}{\gamma'_{n'}} |\mathcal{M}|^2 \\
& \times \delta\left(\sum_{i'=1}^{n'} p^{*'}_{i'} - \sum_{i=1}^{n} p^*_i \right) (f'_1 \dots f'_{n'} - f_1 \dots f_n)
\end{aligned}
$$

Vector potentials drop out. FROM INTEGRALS
Practical simplifications due to scaling of all masses by the same factor S.



Early high-energy processes,
$\sqrt{s} - \sqrt{s_{min}} \gtrsim \delta_c = 1.8$ GeV: production only,
$2 \to N$. Longitudinal phase-space model.

$$I \propto \prod_{j=1} \frac{d\mathbf{p}'_j}{\gamma'_j} \underbrace{\mathrm{e}^{-B\,E'_{\perp j}}}_{\text{TRANSVERSE WEIGHT}} \overset{\text{LONGITUDINAL WEIGHT}}{W_{\|j}} \times \delta\left(p_1 + p_2 - \sum_{j=1}^{N} p'_j\right)$$

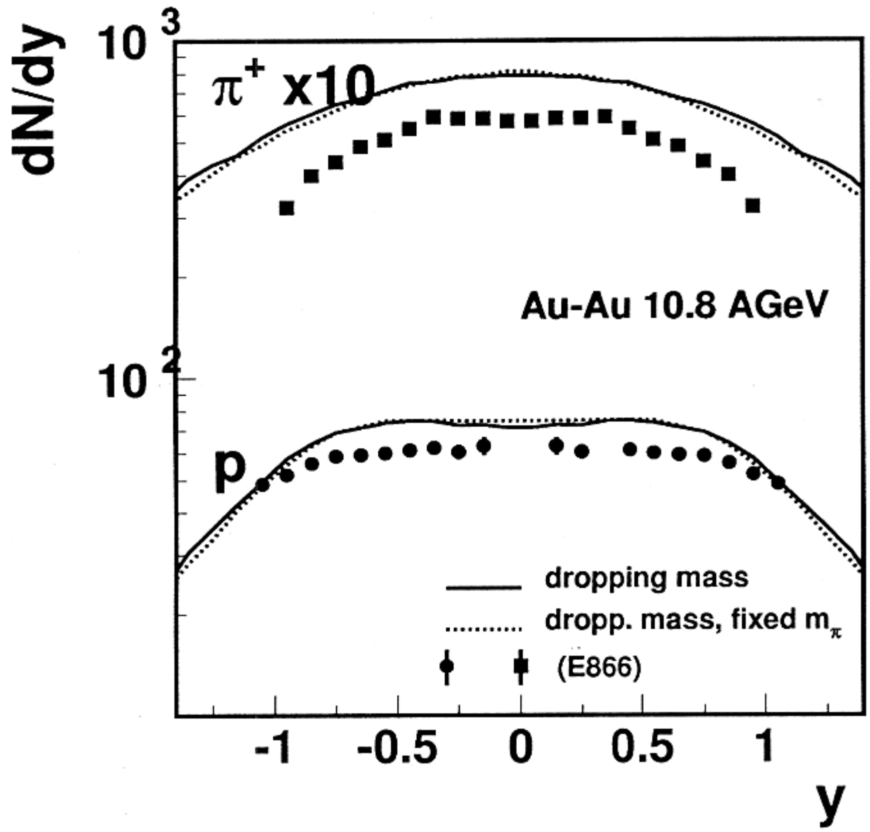
$W_{\|} = \mathrm{e}^{-|y-y_i|}$ for leading ptcles, and $W_{\|} = 1$ for central

Later lower-energy processes treated preserving detailed balance, $2 \leftrightarrow 2$, $2 \leftrightarrow 1$:

elastic, $\pi + N \leftrightarrow \Delta$, $\pi + \Delta \leftrightarrow N + \rho$
$\pi + \pi \leftrightarrow \rho$, $\pi + \pi \leftrightarrow \rho + \rho$, $N + N \leftrightarrow N + \Delta$
$N + N \leftrightarrow \Delta + \Delta$, $N + \Delta \leftrightarrow \Delta + \Delta$, $B + \overline{B} \leftrightarrow \pi + \pi$
$B + \overline{B} \leftrightarrow \rho + \rho$, $B + \overline{B} \leftrightarrow \rho + \pi$

BARYON-RICH REGIME

Rapidity Distributions



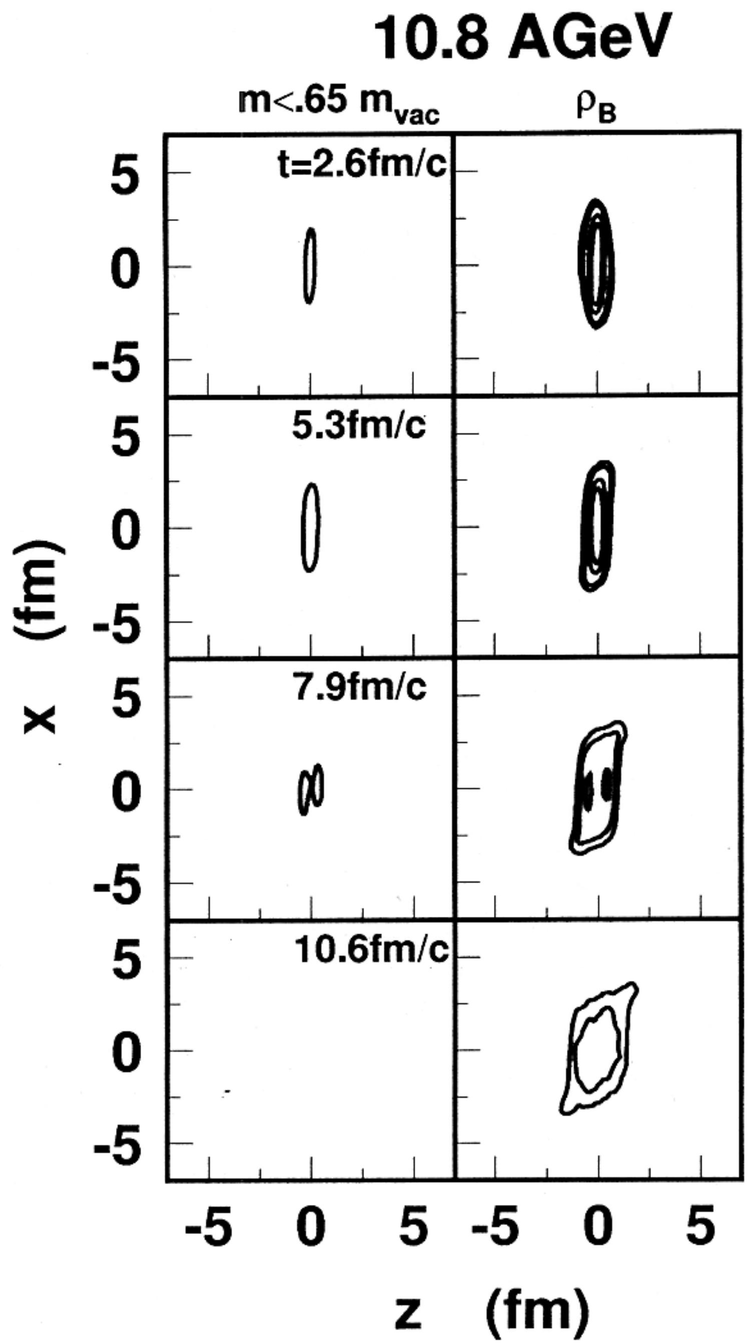
π^+

NOTE: NO K'S

PROTONS

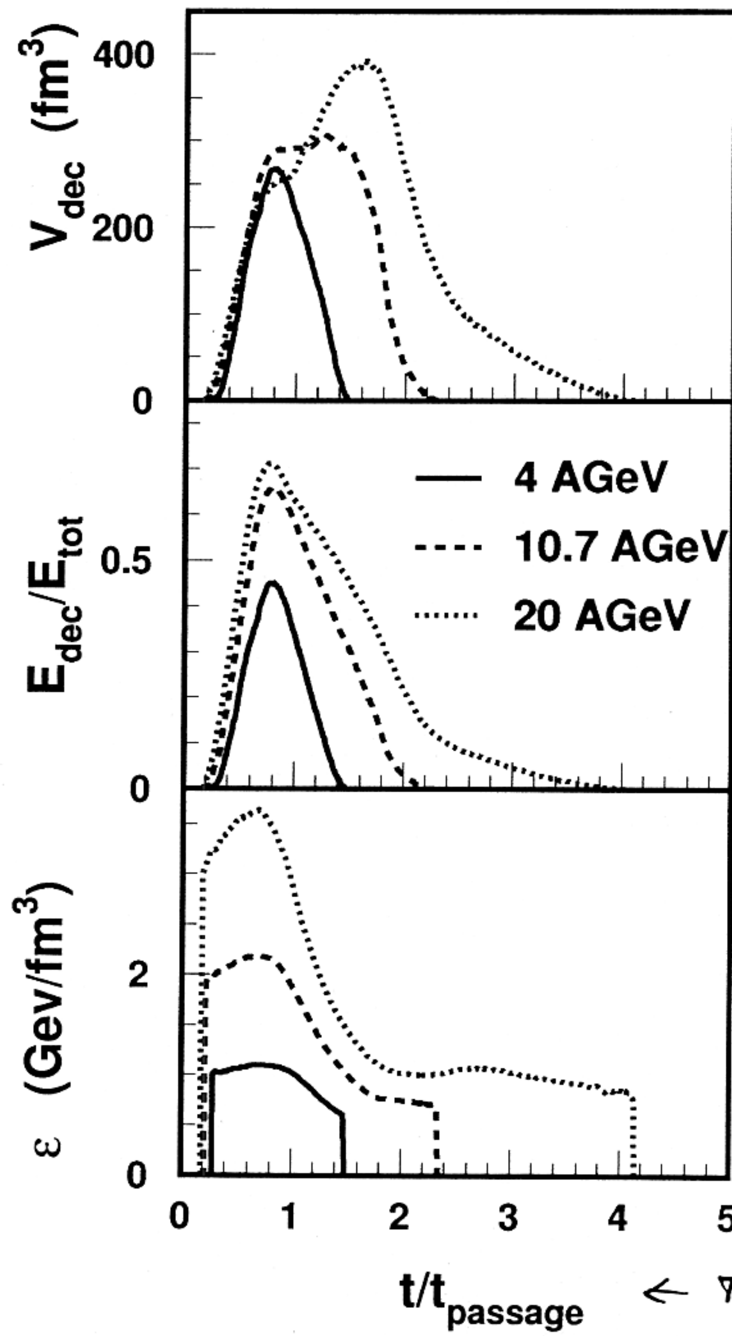
Au + Au

Au + Au Dynamics w/PT



Left – plasma Right – baryon density

Volume & persistence of plasma increase w/inc energy



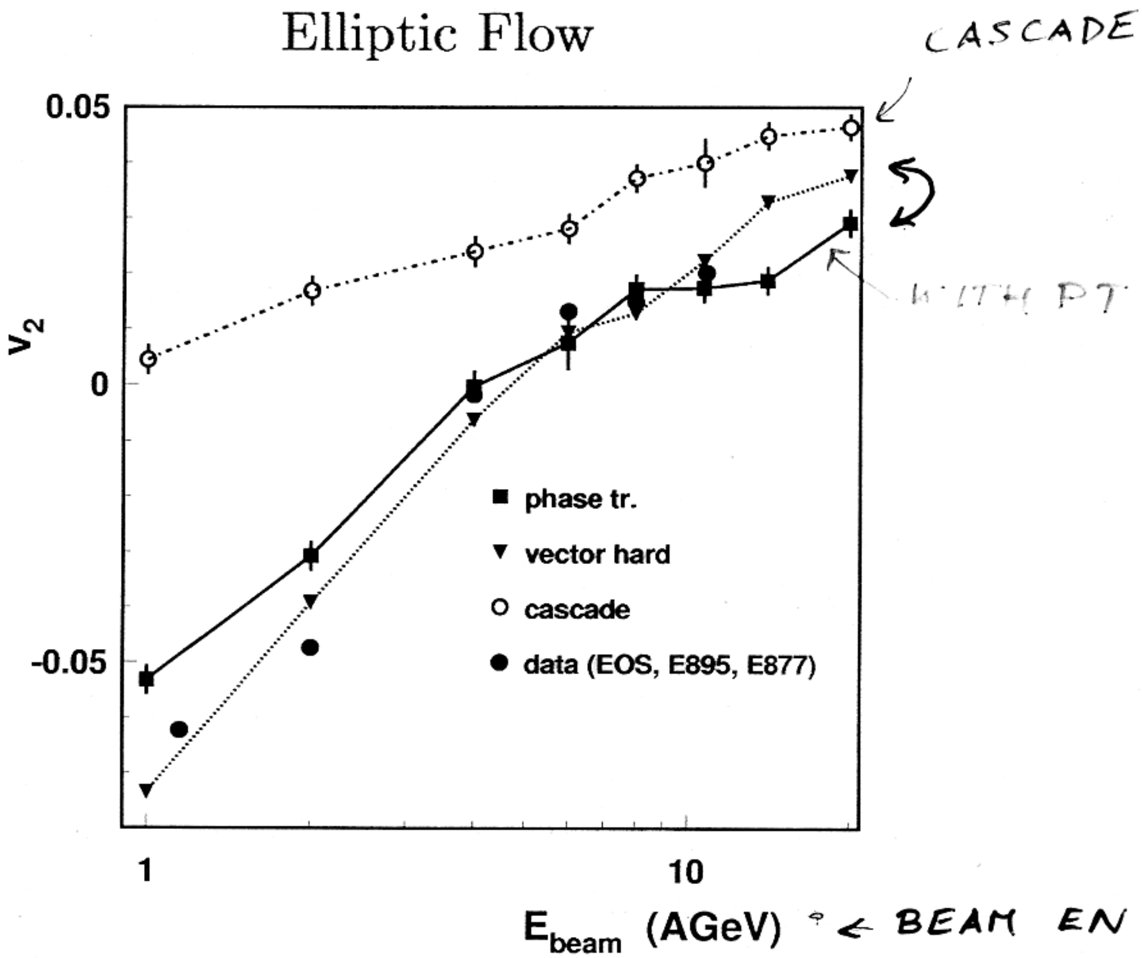
VOLUME OF THE REGION W/ PLASMA

NET ENERGY IN PLASMA TO TOTAL ENERGY

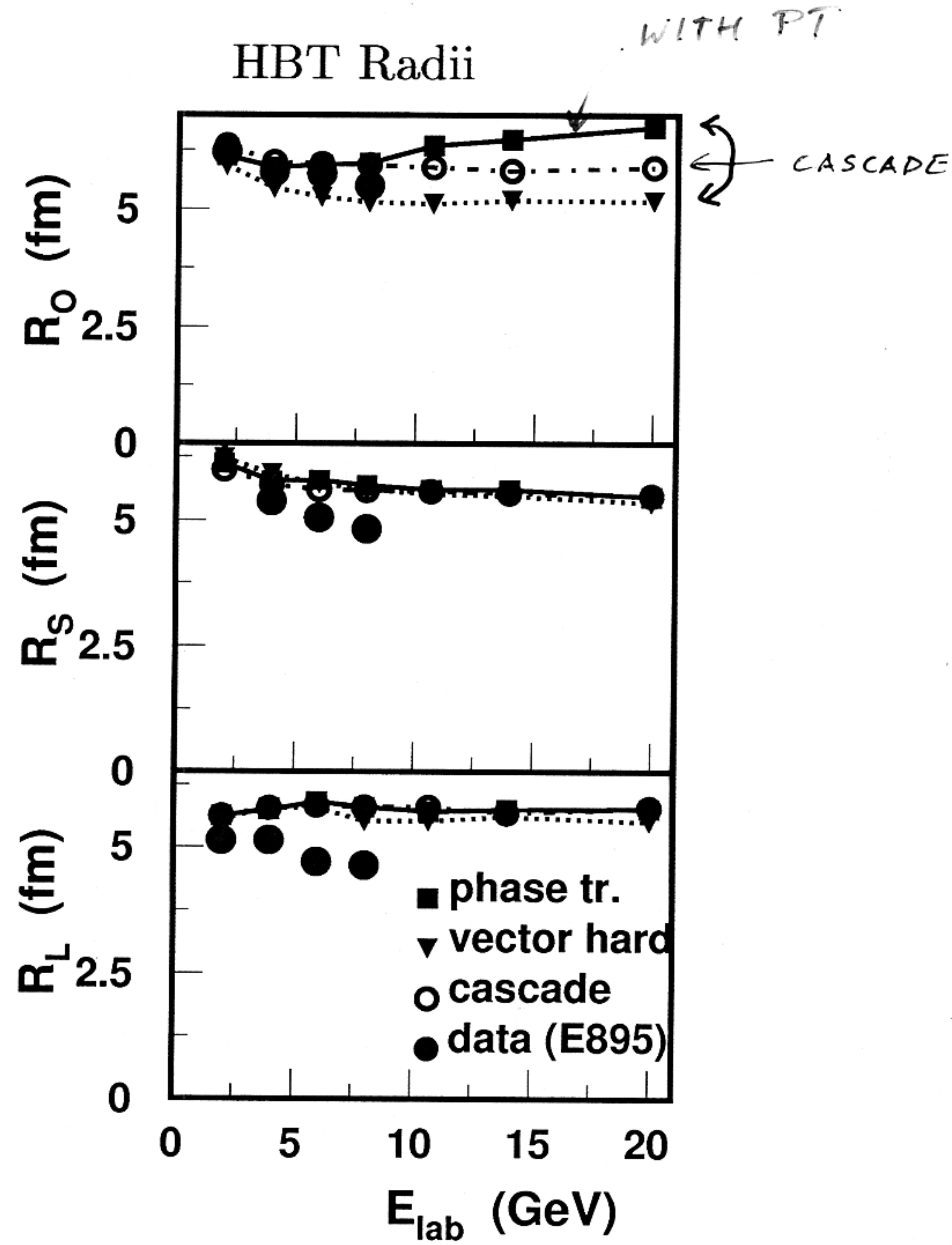
ENERGY DENSITY IN PLASMA

← TIME

Au + Au



Reduction in the v_2-rise as more plasma gets produced.



Increase in R_{out} as more plasma gets produced.

Signals in the excitation functions moderate…

AIMING AT MIDRAPIDITY

Observables at RHIC

High-density initial thermalized state analogous to that in the hydro calcs (Solfrank, Hirano)

$$\epsilon(x\,y\,z) = \frac{N_{WN}(x\,y\,b)}{N_{WN}(0\,0\,0)} \left(\frac{\tau_i}{\sqrt{t^2 - z^2}}\right)^{4/3} \epsilon_0$$

$$n_b(x\,y\,z) = \frac{N_{WN}(x\,y\,b)}{N_{WN}(0\,0\,0)} \left(\frac{\tau_i}{\sqrt{t^2 - z^2}}\right) n_B^0$$

$\sqrt{s} = 130$ AGeV cascade $\epsilon_0 = 19$ GeV/fm^{-3}

$n_B^0 = 0.8$ fm^{-3}

$\rightarrow$ midrap $\frac{dN_{ch}}{d\eta} = 540$

$\frac{dE_\perp}{d\eta} = 630$ GeV

$\tau_i = 1 fm/c$

$\overline{p}/p = 0.58$

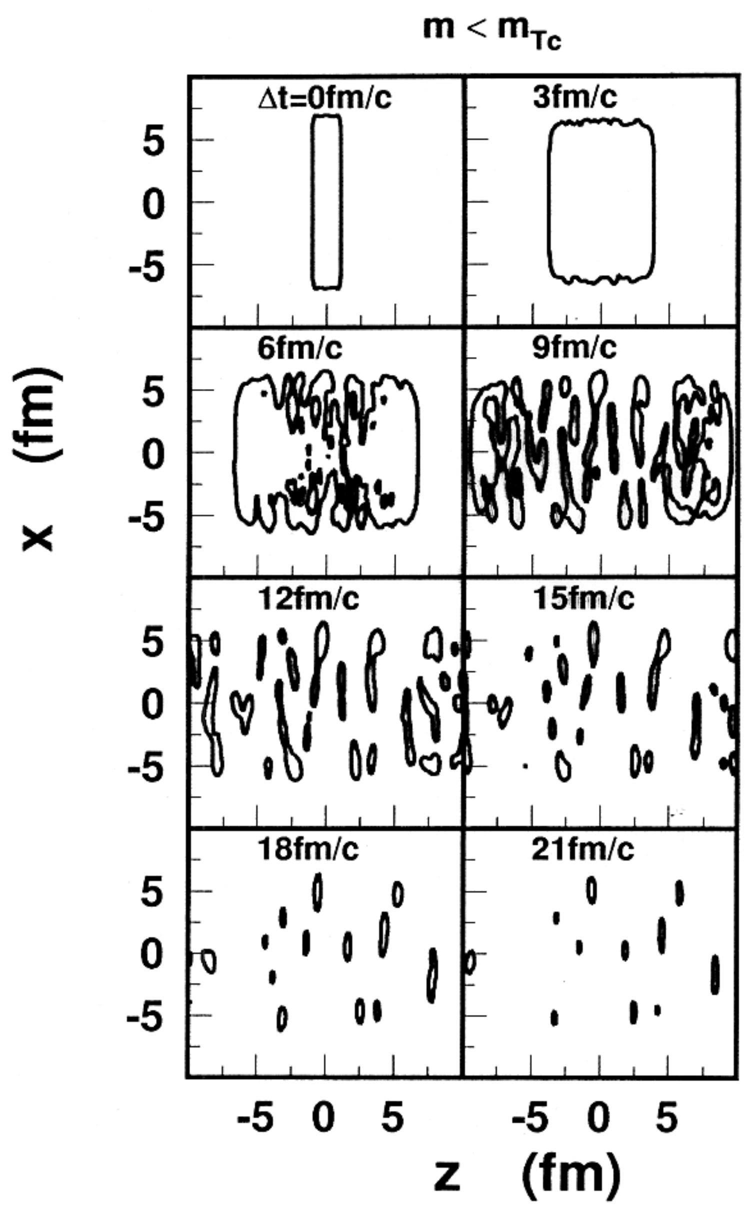
PT $\epsilon_0 = 13$ GeV/fm^{-3}

$n_B^0 = 0.8$ fm^{-3}

$\rightarrow$ midrap $\frac{dN_{ch}}{d\eta} = 720$

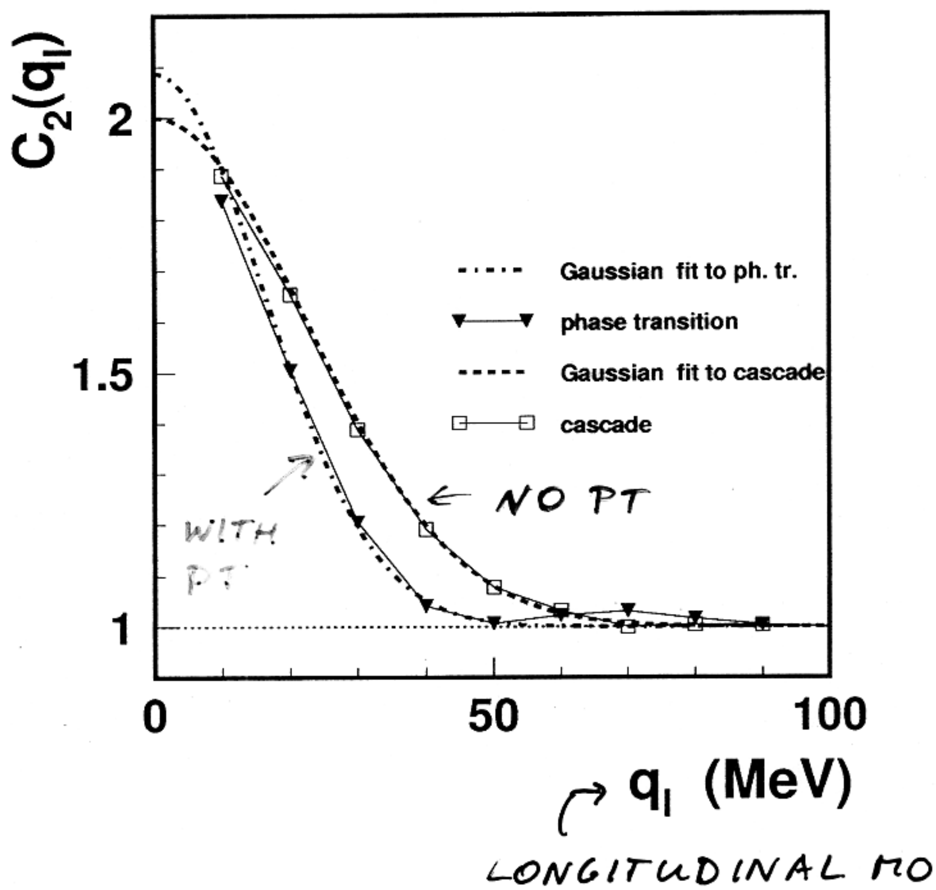
$\frac{dE_\perp}{d\eta} = 590$ GeV

Matter fragments when crossing the phase transition!

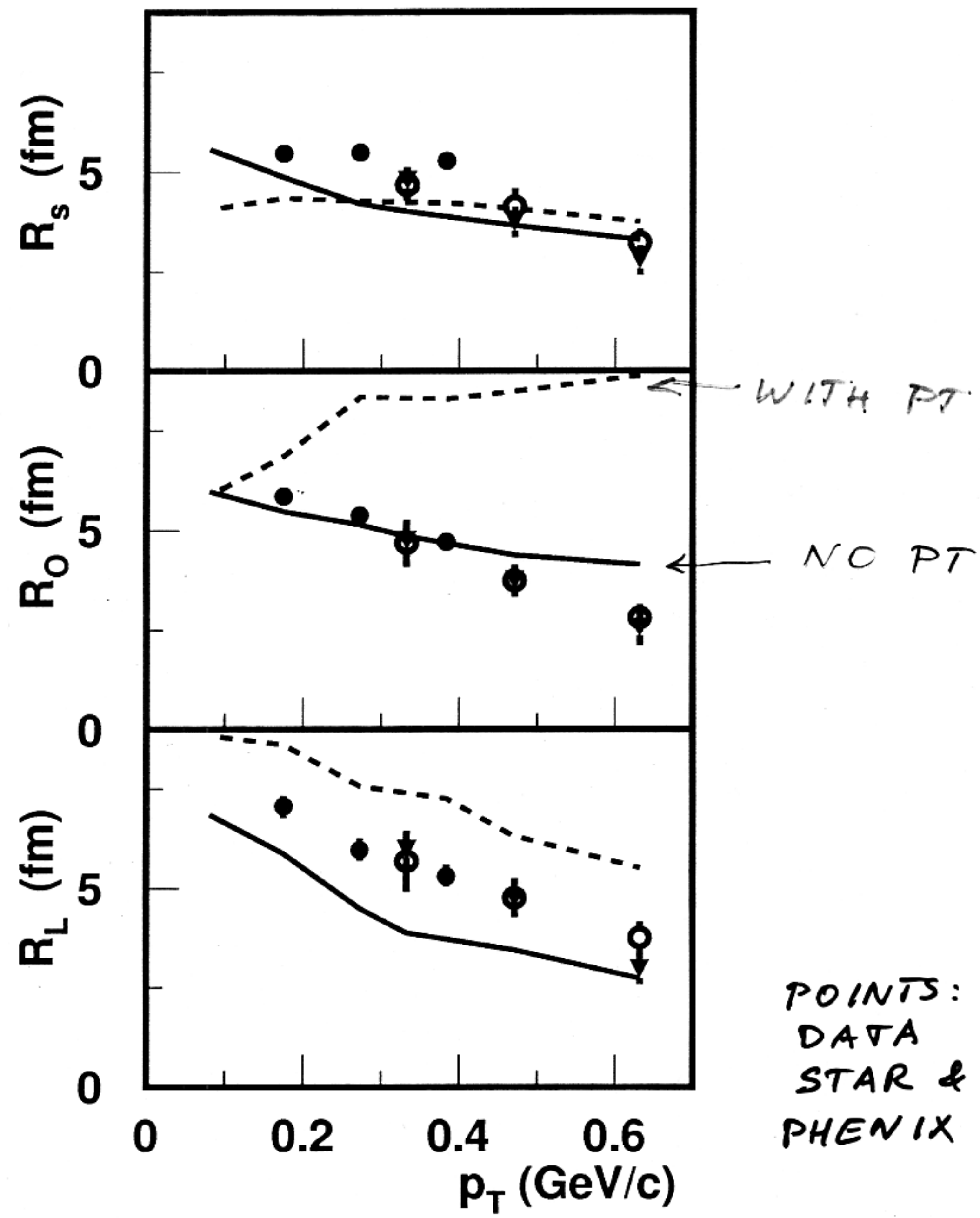


Contours for the high-density phase

Correlation function w/o and w/PT



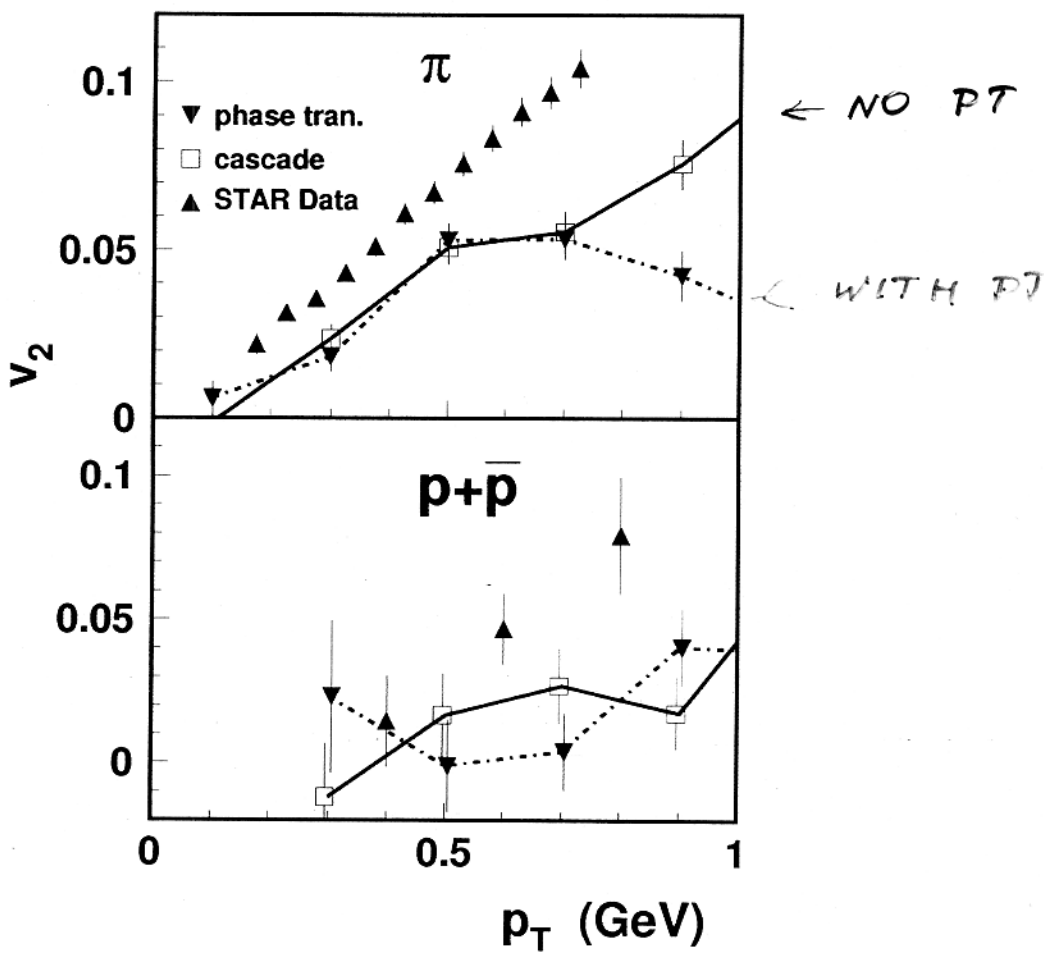
HBT radii w/o and w/PT



Outward radii large for PT

$b = 9.5$ fm Au + Au

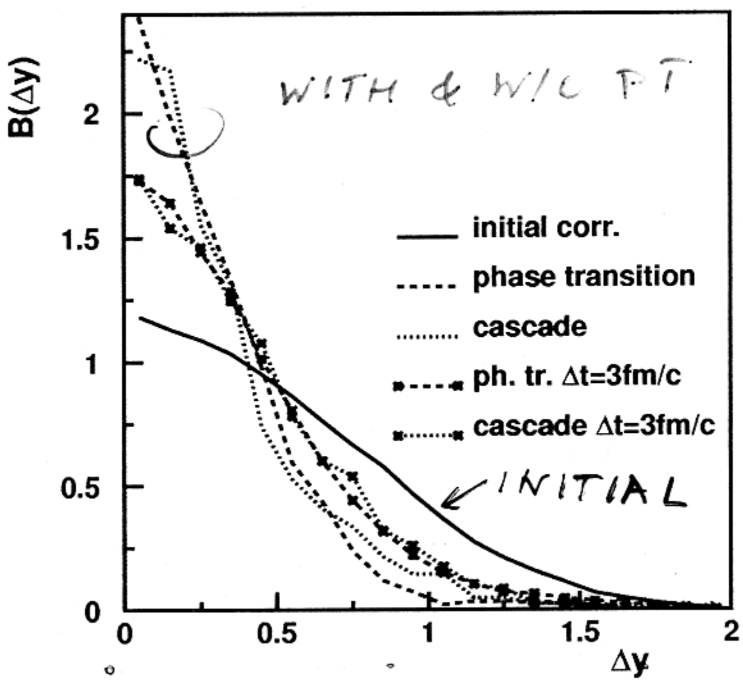
Elliptic Flow



Inconclusive

GENERALLY LOW.
INITIAL CONDITIONS ?

$p - \overline{p}$ Balance Function



RAP Cuts as in STAR $\pi^+ - \pi^-$

Insensitive to PT?...

(but sensitive to initial space-y correlations...)

CONCLUSIONS

- Hadronic transport model: flexibility in incorporating the phase transition, changing thermodynamic properties, variation of transport.
- In the baryon-rich regime, the transition signaled by a moderate simultaneous softening in v_2 and rise in R_{out} in the excitation functions.
- At RHIC, a system going through the 1-order phase-transition fragments into hot domains.
- The transition results in a humongous R_{out}. The data are more consistent with the lack of the transition.