Placing Bounds on Hydrodynamics/Equilibirium:

Perfect Thermal Equilibrium $\ell_{mfp}/L = 0$

$$f_o = \frac{1}{\exp\left(\frac{p\cdot u}{T}\right) - 1}$$

Non-Equilibirium Effects $\ell_{mfp}/L \ll 1$ modify this distribution

- Finite lifetime/size of the system
- Finite cross sections
- Expansion Dynamics – Gradients in Velocity

$$f \to f_o + \delta f$$

Calculate the non-equilibrium corrections δf.

These corrections modify Spectra and $v_2(p_T)$.

Viscous corrections to Ideal Hydrodynamics:

$$T^{\mu\nu} = T_o^{\mu\nu} + T_{vis}^{\mu\nu}$$

Then with this stress tensor we have:

$$\begin{aligned} T_o^{\mu\nu} &= (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \\ T_{vis}^{\mu\nu} &= \eta\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}\delta_{\alpha\beta}\,\partial_\gamma u^\gamma) \\ &\equiv \eta\,\langle\partial^\mu u^\nu\rangle \end{aligned}$$

For a Bjorken expansion we have:

$$T_{vis}^{zz} \sim \eta\partial^z u^z \sim -\frac{\eta}{\tau}$$

Doing the algebra:

$$\begin{aligned} T^{\mu\nu} &= T_o^{\mu\nu} + T_{vis}^{\mu\nu} \\ &= \begin{pmatrix} \epsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} + \begin{pmatrix} 0 & & & \\ & \frac{2}{3}\frac{\eta}{\tau} & & \\ & & \frac{2}{3}\frac{\eta}{\tau} & \\ & & & -\frac{4}{3}\frac{\eta}{\tau} \end{pmatrix} \end{aligned}$$

- The Longitudinal Pressure is reduced by $\frac{4}{3}\eta/\tau$.
- The Transverse Pressure is increased by $\frac{2}{3}\eta/\tau$.

Expect p_T spectra to be pushed out to larger p_T.

How big is ℓ_{mfp}/L? How much Entropy is produced?

$$\frac{d(\tau s)}{d\tau} = 0 \qquad \text{(Ideal Case)}$$
$$\frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T} \qquad \text{(Viscous Case)}$$

For hydrodynamics to be valid, the entropy produced over the time scale of the system, τ, must be small compared to the total :

$$\tau \frac{\frac{4}{3}\eta}{\tau T} \frac{1}{\tau s} \equiv \Gamma_s \frac{1}{\tau} \ll 1 \tag{1}$$

where Γ_s is the *Sound Attenuation Length.*

$$\Gamma_s \equiv \frac{\frac{4}{3}\eta}{sT} \rightarrow \text{Units of Fm}$$

<u>Some Estimates:</u>

- In QGP Phase: $\tau \sim 3$ Fm and $\Gamma_s \sim 1$ Fm.
- In Hadron Phase: $\tau \sim 9$ Fm and $\Gamma_s \sim 3$ Fm.

Below we take the hydrodynamic expansion parameter, $\frac{\Gamma_s}{\tau}$, to be $\frac{1}{3}$. If we are lucky then $\frac{\Gamma_s}{\tau}$ is $\frac{1}{6}$. If unlucky then there is no Hydro.

Viscous corrections to the distribution function, δf

General Features:

- δf is proportional to the strains:
  $\langle \partial_\mu u_\nu \rangle, \cancel{\partial_\gamma u^\gamma}, \cancel{\partial_\gamma T}, \cancel{\partial_\gamma \mu}$
- δf is a scalar $\delta f \propto \chi(p) p^\mu p^\nu \langle \partial_\mu u_\nu \rangle$.
- If I restrict $f(p) = f_o(1 + \mathrm{g(p)})$ where g(p) is a polynomial of degree less than two. Then the form is completely determined:

$$f = f_o\left(\frac{p \cdot u}{T}\right)\left(1 + \frac{C}{T^3} p_\mu p_\nu \langle \partial^\mu u^\nu \rangle\right)$$

The constant $\frac{C}{T}$ is essentially the sound attenuation constant:

$$\begin{aligned} T^{\mu\nu} &= \int d^3p \, \frac{p^\mu p^\nu}{E} \, f \\ T_o^{\mu\nu} + T_{vis}^{\mu\nu} &= \int d^3p \, \frac{p^\mu p^\nu}{E} \, (f_o + \delta f) \end{aligned}$$

Then looking only at the viscous piece:

$$T_{vis}^{\mu\nu} = \eta \langle \partial^\mu u^\nu \rangle = \underbrace{\int d^3p \, \frac{p^\mu p^\nu}{E} \, f_o \, \frac{C}{T^3} p_\alpha p_\beta}\, \langle \partial^\alpha u^\beta \rangle$$

- Massless Gas: C = 1.06 $\frac{\eta}{s}$
- Non-Relativistic Gas: C = 0.94 $\frac{\eta}{s}$

Formal procedure to determine δf (see de Groot) :

- Write $f = f_o(\frac{p \cdot u}{T}) + \delta f$ and substitute into the Boltzmann Equation:

$$\partial_t f + \vec{p} \cdot \nabla_x f = C[f]$$

- Keep terms proportional to first powers of the gradient.

$$\partial_t f_o + \vec{p} \cdot \nabla_x f_o = L[\delta f]$$

- This is an Linear integral equation for δf. If you know the cross sections then you can solve this integral equation.

Both in the perturbative QGP phase (G. Baym) and the hadron gas (R. Venugopalan) an ansatz of the following form is good approximation to the numerical results:

$$f = f_o + f_o(1 + f_o)\frac{C}{T^3} p_\mu p_\nu \left\langle \partial^\mu u^\nu \right\rangle \qquad (2)$$

This amounts to taking a relaxation time which grows linearly with p_T

Spectra – Bjorken Cylinder:

$$\frac{d^2N}{d^2p_T\,dy} = \int p^\mu d\Sigma_\mu\, f$$
$$dN_o + \delta\, dN = \int p^\mu d\Sigma_\mu\, f_o + \delta f$$

Want to Compute $\frac{\delta\, dN}{dN_o}$**:**

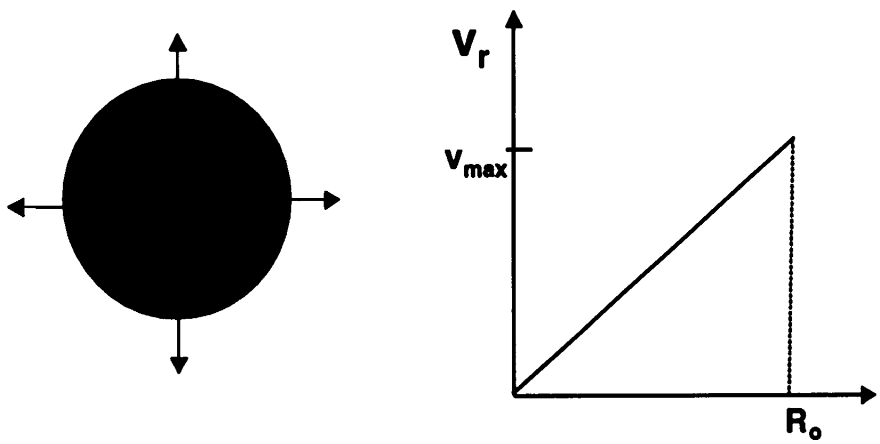
$$\delta f \approx f_0 \Gamma_s \frac{p_\alpha}{T}\frac{p_\beta}{T}\left\langle \partial^\alpha u^\beta \right\rangle \sim f_0 \Gamma_s \left(\frac{p_T}{T}\right)^2 \frac{1}{\tau}$$

Now you can do these integrals:

$$\frac{\delta\, dN}{dN_o} = \frac{\Gamma_s}{2\tau}\left\{\left(\frac{p_T}{T}\right)^2 - \left(\frac{m_T}{T}\right)^2 \frac{1}{2}\left(\frac{K_3(\frac{m_T}{T})}{K_1(\frac{m_T}{T})} - 1\right)\right\}$$
$$\to \frac{\Gamma_s}{2\tau}\left(\frac{p_T}{T}\right)^2$$

- **When viscous corrections become of order one, then we are supposed to throw away hydrodynamics.**
- **This puts a bound on how high in p_T the hydrodynamic description may be applied.**

Spectra with transverse expansion :



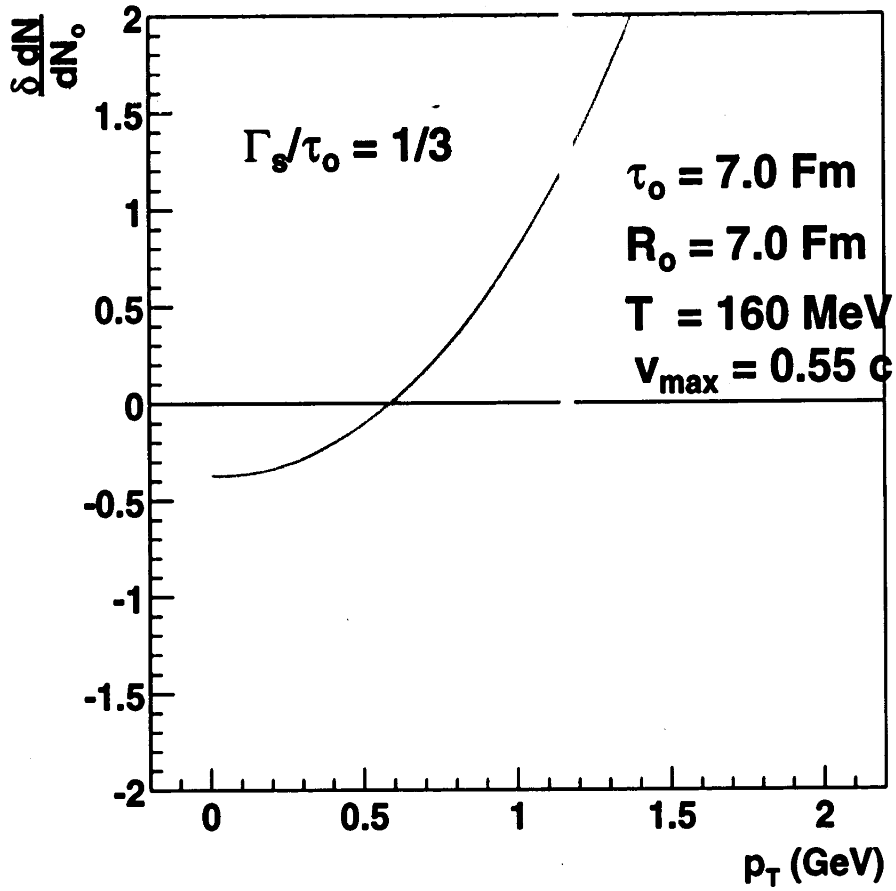
Compute $\langle \partial^\mu u^\nu \rangle$:

- Now Find two gradients: $\frac{1}{\tau}$ and $\frac{v_{max}}{R_o}$
- Compute the correction to the spectrum. Each term in $p_\mu p_\nu \langle \partial^\mu u^\nu \rangle$ gives a couple of Bessels.
- For large p_T

$$\frac{\delta\, dN}{dN_o} = \frac{\Gamma_s}{2\tau}\left(\frac{p_T}{T_{eff}}\right)^2 \left(\frac{1}{\tau} - \frac{v_{max}}{R}\right)$$

where

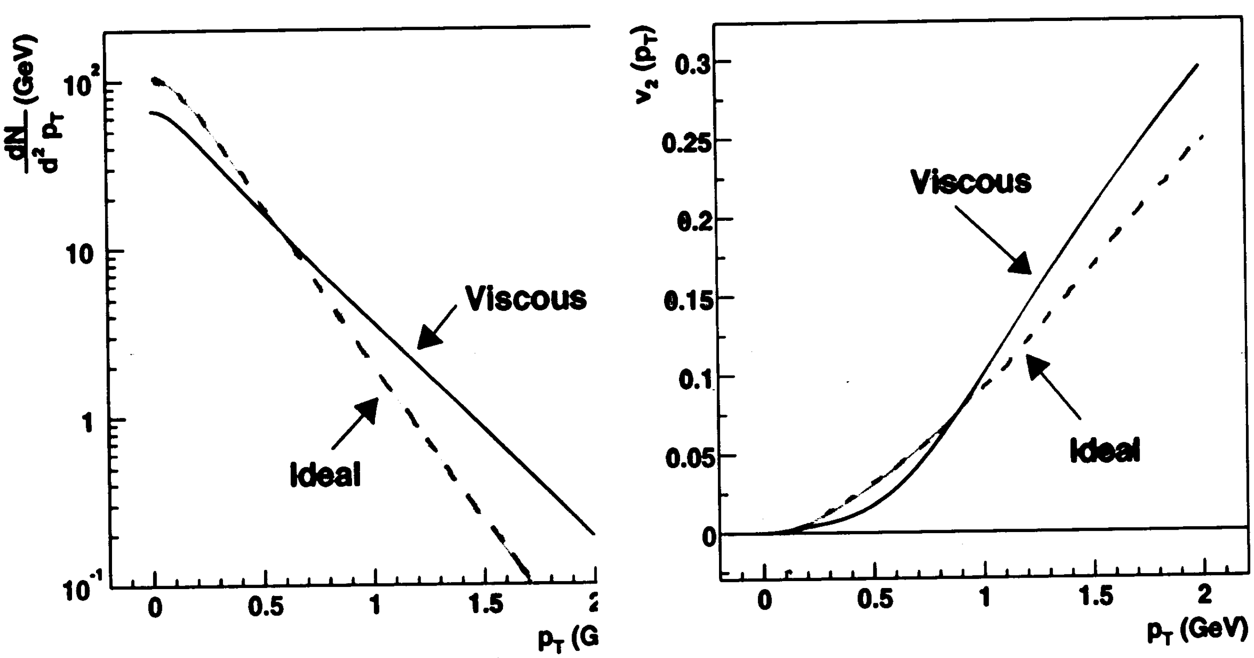
$$T_{eff} = T\sqrt{\frac{1+v}{1-v}}$$



The maximum possible p_T accessible to Hydrodynamics is 1.0-1.3 GeV – A couple of times T_{eff} .

V_2 of the Viscous Blast Wave

- Take parameters from the Ideal Blast Wave Fit (With no spatial anistropy $s_2 = 0$).
- Then with the parameters fixed calculate the viscous correction



V_2 is reduced at small p_T. Shear Viscosity quenches Elliptic flow.

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