

# How The HBT Puzzle Might Dissipate

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BNL Workshop 6/14/16

- Numerical Results from ideal Hydro + Cascade
- Analytical Computation to First-Order in Deviation from Local Equilibrium: Qualitative Insight into "Freeze-Out" of Single-Inclusive & HBT Correlations
- Why the Dynamics of Non-Equilibrium Phase Transitions Matters!

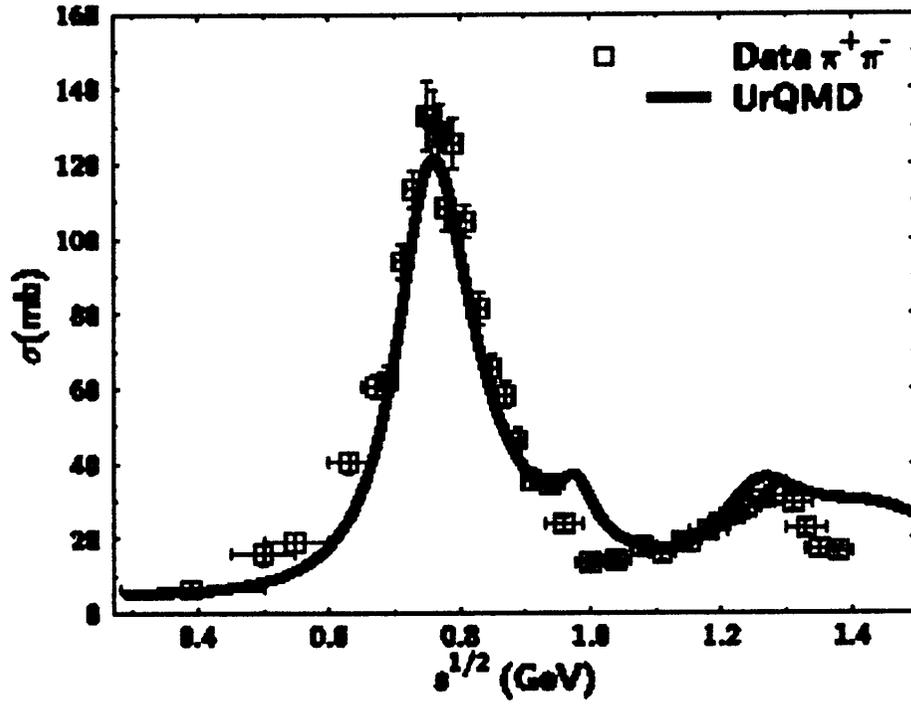


FIG. 11. The total cross-section of  $\pi^+\pi^-$  scattering as a function of c.m. energy  $\sqrt{s}$ . Data (open squares) are taken from [19].

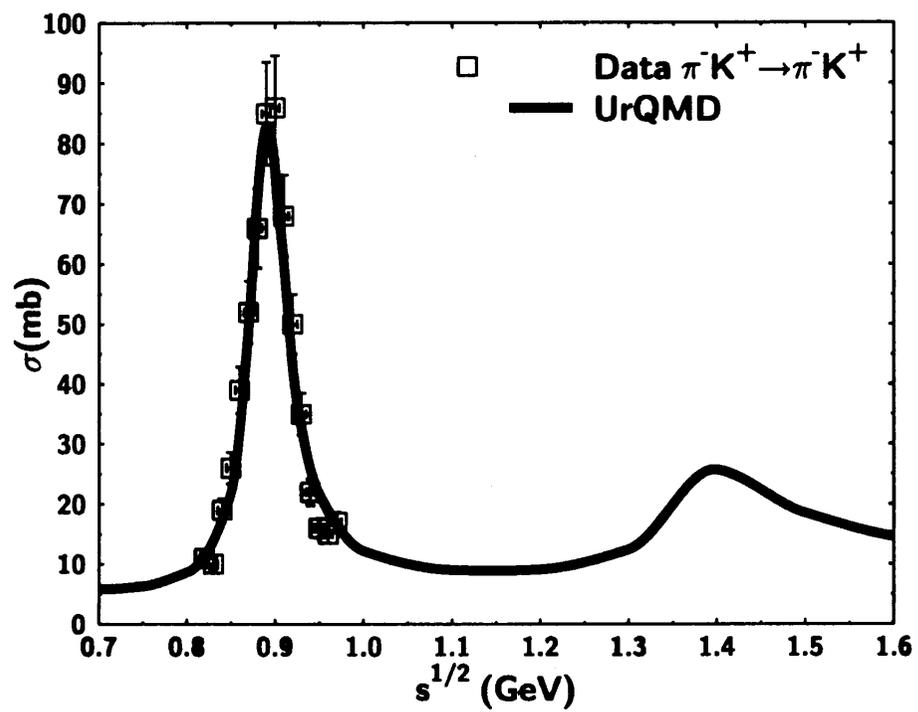


FIG. 14. Cross-section of  $\pi^- K^+$  scattering vs.  $\sqrt{s}$ . Data (open squares) are taken from [22].

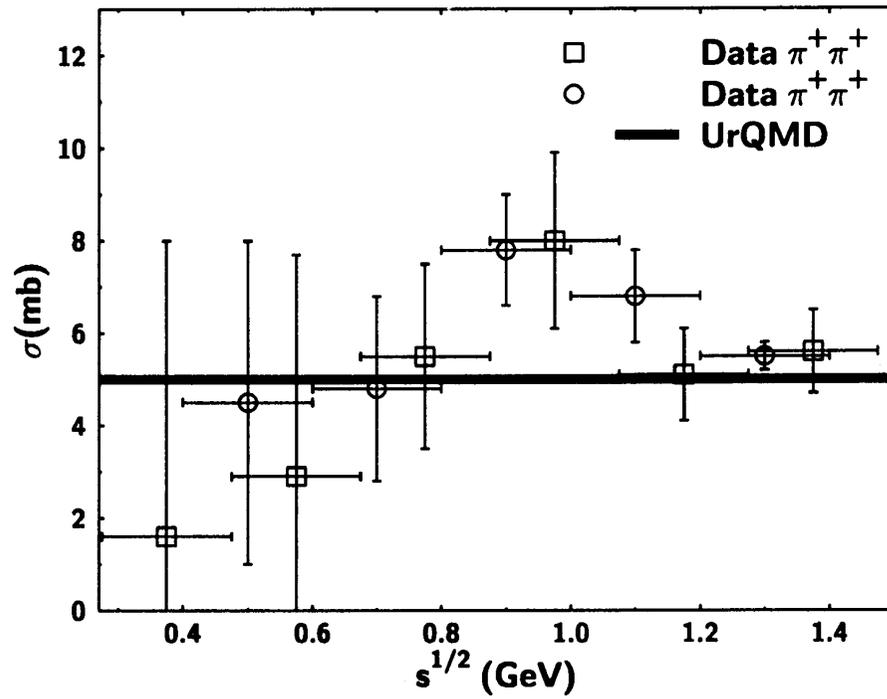


FIG. 12. The same as Fig. 11 but for  $\pi^+\pi^+$  scattering. Data are taken from [20] (open squares) and from [21] (open circles).

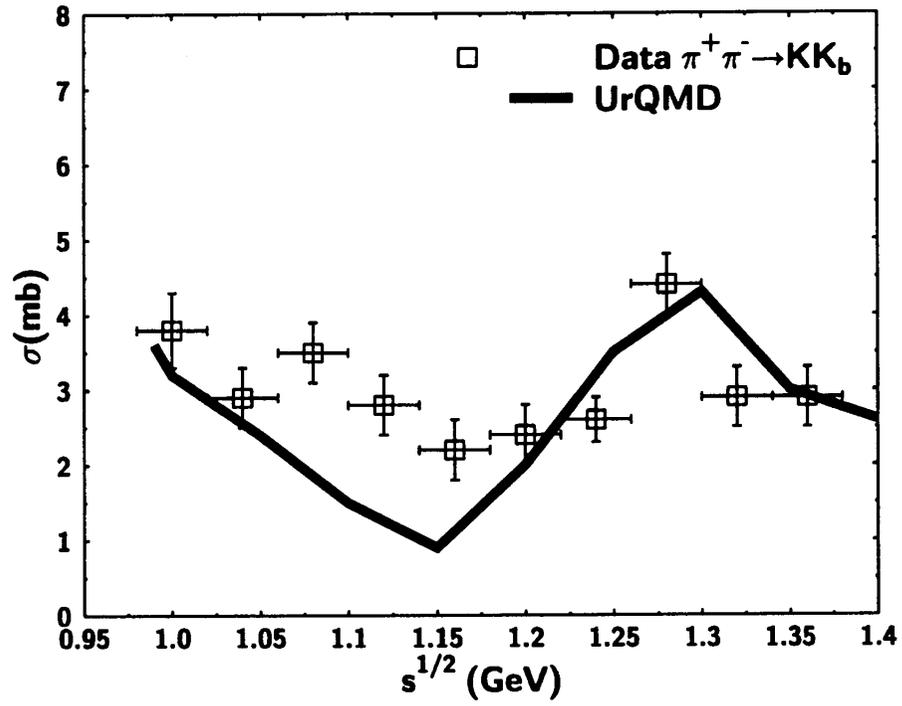


FIG. 13. Cross-section of the reaction  $\pi^+\pi^- \rightarrow K\bar{K}$  as a function of  $\sqrt{s}$ . Data (open squares) are taken from [19].

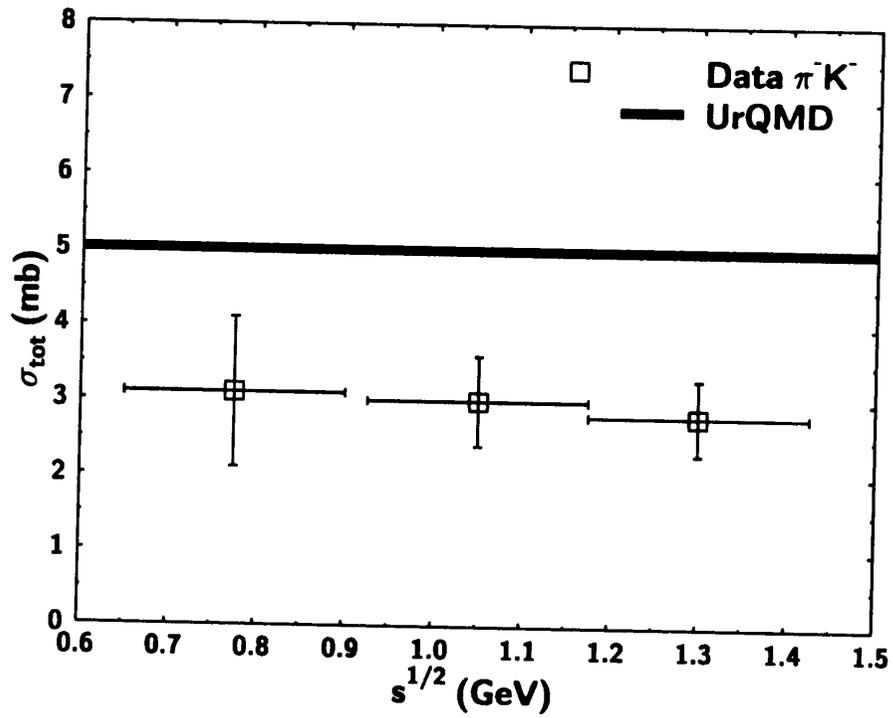
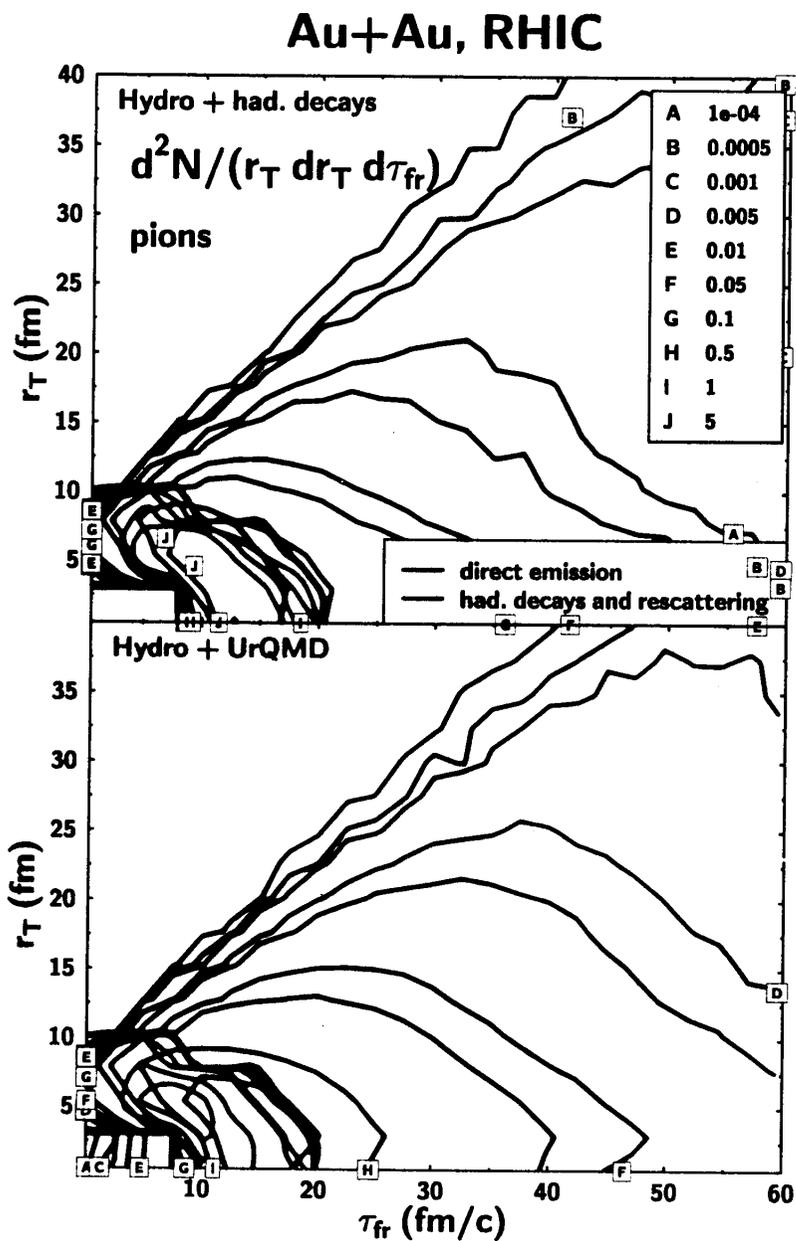


FIG. 15. The same as Fig. 14 but for  $\pi^- K^-$  reaction. Data (open squares) are taken from [23].

# Pion freeze-out hypersurface

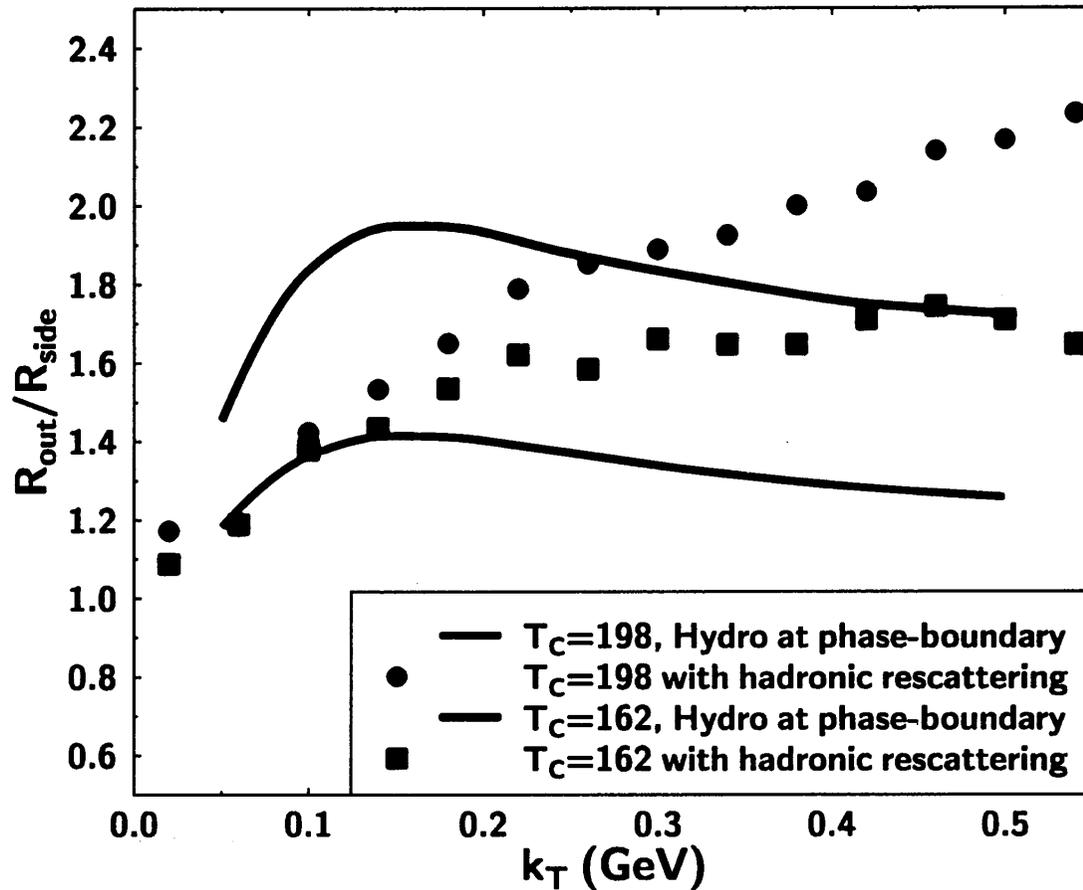


- source extended both in temporal and radial direction due to rescattering

→ sensitivity towards HBT analysis

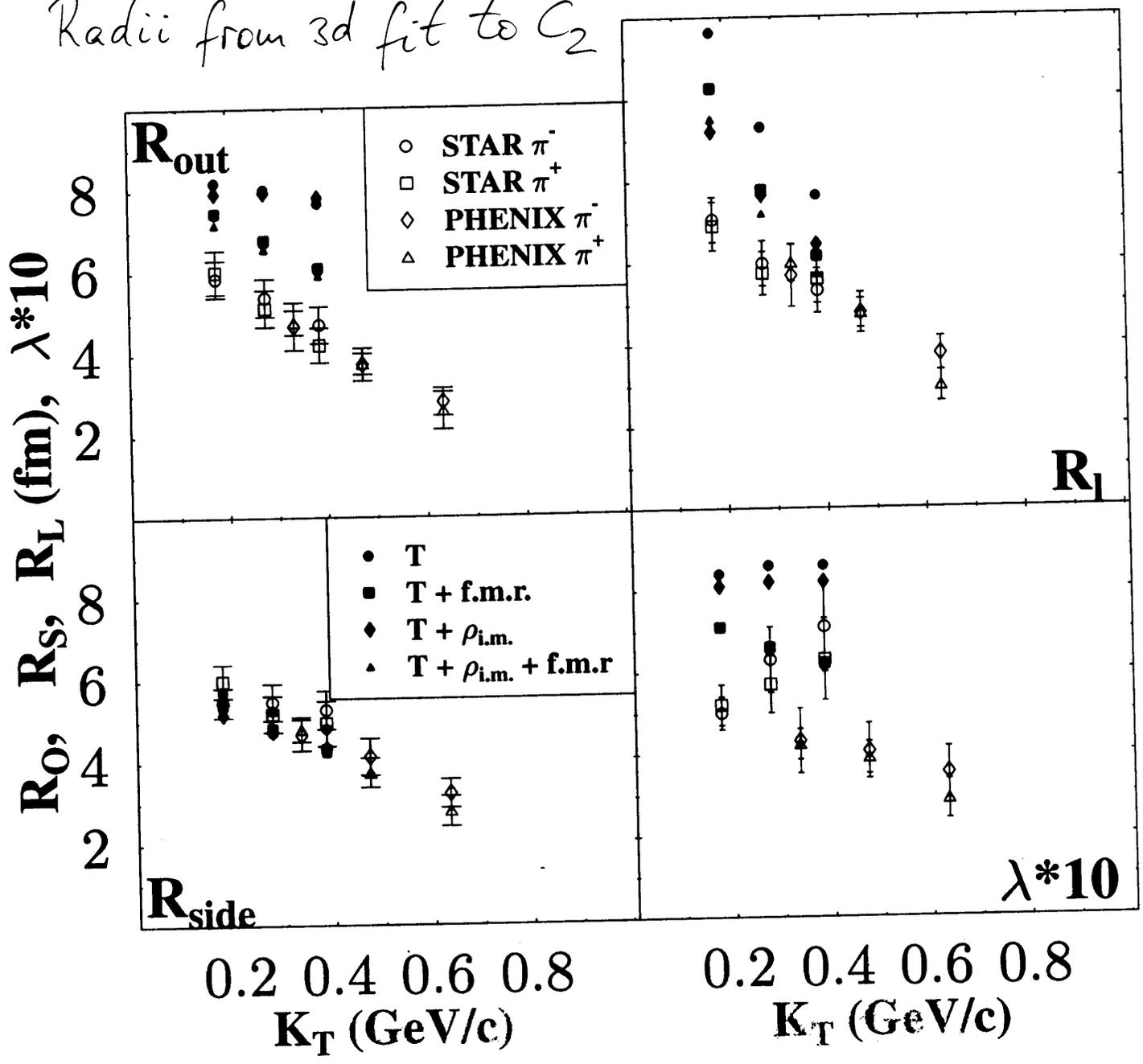
# $R_{\text{out}}/R_{\text{side}}$ at Freeze-out

## Au+Au @ RHIC



- ∴ Dependence on  $T_c$  is reversed with hadronic rescattering
- ∴ Higher  $T_c$  speeds up hadronization but prolongs dissipative hadronic phase (dominating the HBT-radii)

Radii from 3d fit to  $C_2$



"Optimistic"  $T_c = 160$  MeV

# First-Order Corrections to Local Equilibrium

Boltzmann Eq:  $p \cdot \partial f(x^r, p^r) = G[f(x^r, p^r)]$

Global Equilibrium:  $f(x^r, p^r) = f_{eq}(\frac{E}{T})$  ok since  $G[f_{eq}] = 0$

Local Equilibrium:  $f(x^r, p^r) = f_{eq}(E(x^r), T(x^r))$  is v.t. ... l. t. i. o. n.

→  $f = f_{eq} + \delta f$

$$p \cdot \partial (f_{eq} + \delta f) = G[f_{eq} + \delta f] = \underbrace{G[f_{eq}]}_{=0} + \delta f \underbrace{\frac{\delta G}{\delta f}}_{f_{eq}} + \mathcal{O}(\delta f^2)$$

drop

$\equiv -\frac{E}{T_c}$

$$\delta f = -\frac{T_c}{E} p \cdot \partial f_{eq}(E(x^r), T(x^r))$$

$$= \frac{T_c}{E} f_{eq} p \cdot \partial \frac{p \cdot u(x^r)}{T(x^r)} \quad \text{for Boltzmann Distribution}$$

$(m_{\perp} \gg T)$

1d Boost-Invariant Expansion:

$$\eta_f = \text{Arctanh}(V_{||}) \equiv \eta = \text{Arctanh}\left(\frac{x}{t}\right)$$

# Single Inclusive Distribution

$$\tilde{N}(p) = \frac{dN}{d^2 p_{\perp} dy} = \int d\sigma \cdot p f(\sigma, \frac{p \cdot y}{T}) = \int d\sigma \cdot p \left\{ f_{eq} + \delta f \right\}$$

$$= \pi R^2 \tau \sqrt{2\pi T m_{\perp}} e^{-m_{\perp}/T} \left\{ 1 + \gamma \frac{\tau_c}{\tau} \frac{m_{\perp}}{T} \right\}$$

$$\gamma \equiv - \frac{\partial \log T}{\partial \log \tau} \sim \frac{1}{4} - \frac{1}{3}$$

$$T \sim \frac{1}{\tau}$$

## "Freeze-Out"

$$\frac{d\tilde{N}(p)}{d\tau} \stackrel{!}{=} 0 \rightarrow \dot{\tau}_c = 1 + \gamma \frac{m_{\perp}}{T} \frac{\tau_c}{\tau} - \frac{\gamma}{2} \frac{\tau_c}{\tau} \quad (m_{\perp}/T \gg 1)$$

i) start with perfect fluid:  $\tau_c(\tau_0) = 0$  ( $\tau_0$  = hadronization time, or later)

$$\tau_c = \tau - \tau_0 \quad (\text{as long as } \tau_c \ll \tau)$$

Linear Increase of Relaxation Time required

Reasonable, think of  $\sigma = \text{const}$ ,  $\tau_c = \frac{1}{\sigma \epsilon}$

ii) very dissipative fluid:  $\tau_c \sim \tau$

$$\frac{\tau_c}{\tau_0} \sim \sqrt{\frac{T}{T_0}} \exp\left\{ \frac{m_{\perp}}{T} - \frac{m_{\perp}}{T_0} \right\}$$

Non-linear growth required  $\rightarrow$  breakdown of first-order theory

# Two-Particle Correlations

$$C_2(p_1, p_2) - 1 = \frac{1}{\tilde{N}(p_1) \tilde{N}(p_2)} \left| \int d\sigma \cdot k e^{i\sigma \cdot q} f\left(\frac{u \cdot k}{T}\right) \right|^2$$

$$f = f_{eq} + \delta f$$

$$k = \frac{1}{2}(p_1 + p_2), \quad q = p_1 - p_2$$

For  $\alpha \equiv y_2 - y_1 \ll 1$  but  $\alpha \tau T, \alpha \frac{m_\perp}{T} = O(1)$ :

$$C_2(\alpha, m_\perp) - 1 = \cos^2(\alpha \tau T) e^{-\alpha^2 m_\perp \tau^2 T}$$

$$\times \left\{ 1 - \frac{\tau_c}{\tau} \frac{m_\perp}{T} \frac{\alpha^2}{2} \left[ 1 - \gamma - \xi_T + 2 \Delta_T^2 \right] \right\}$$

$$\gamma = - \left\langle \frac{\partial \log T}{\partial \log \tau} \right\rangle$$

$$T \sim \frac{1}{\tau \sigma}$$

$$\xi_T = \frac{1}{T} \left\langle \frac{\partial^2 T}{\partial y^2} \right\rangle$$

curvature of  $T$  in rapidity

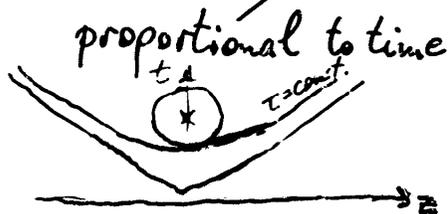
$$\Delta_T^2 = \left\langle \left( \frac{1}{T} \frac{\partial T}{\partial y} \right)^2 \right\rangle$$

mean-square  $T$  fluctuation

Longitudinal Homogeneity Scale:

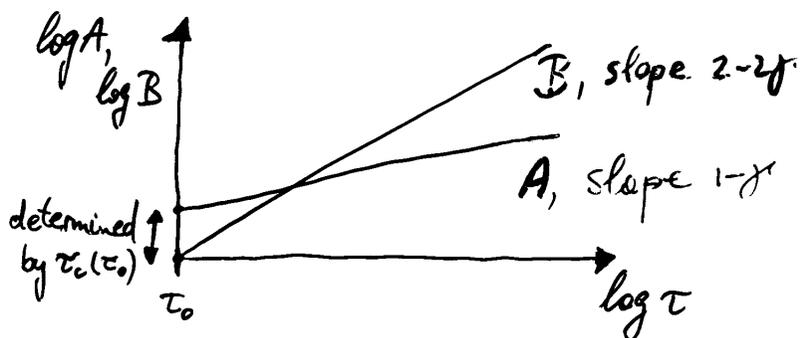
$$R_{||} = \left\{ -\frac{1}{2} \frac{1}{m_\perp^2} \frac{\partial^2 C_2(\alpha)}{\partial \alpha^2} \right\}^{1/2} = \tau \sqrt{\frac{T}{m_\perp}} \sqrt{1 + \frac{\tau_c}{\tau} \frac{1 - \gamma - \xi_T + 2 \Delta_T^2}{2 \tau^2 T^2}}$$

scales  $\sim \frac{1}{\sqrt{m_\perp}}$



# "Freeze-Out" of $R_{II}$ :

$$\frac{dR_{II}}{d\tau} \stackrel{!}{=} 0 \rightarrow \tau_c = \underbrace{(1-\gamma) \frac{\tau_c}{\tau}}_{A(\tau)} - 2 \underbrace{\frac{2-\gamma}{1-\gamma} \tau^2 T^2}_{B(\tau)}$$



- $\tau_c$  is not positive for  $\tau \rightarrow \infty$   
OR  
 $R_{II}$  doesn't freeze out ...

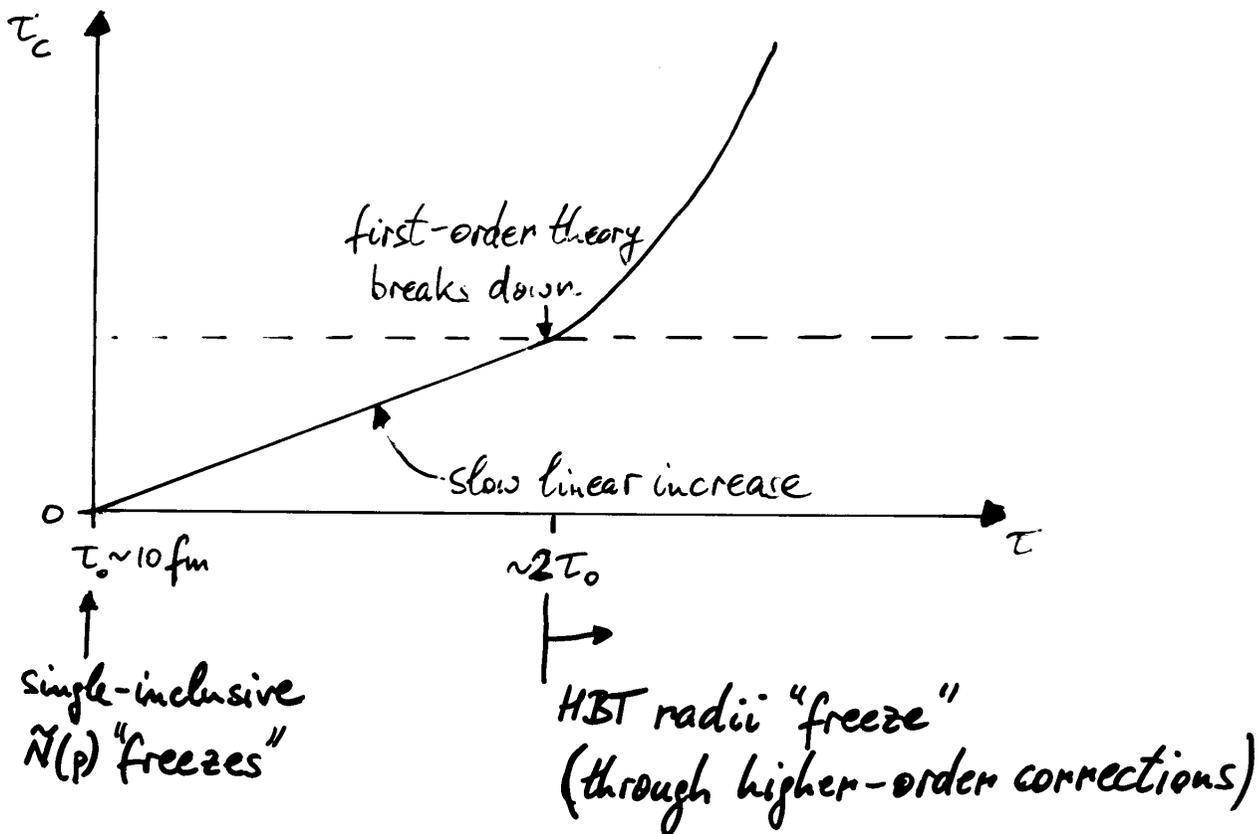
Either of which seems unphysical.

- Emergency Evacuation Route:

Corrections of Higher Order, that is:

Hadron Fluid far from Equilibrium

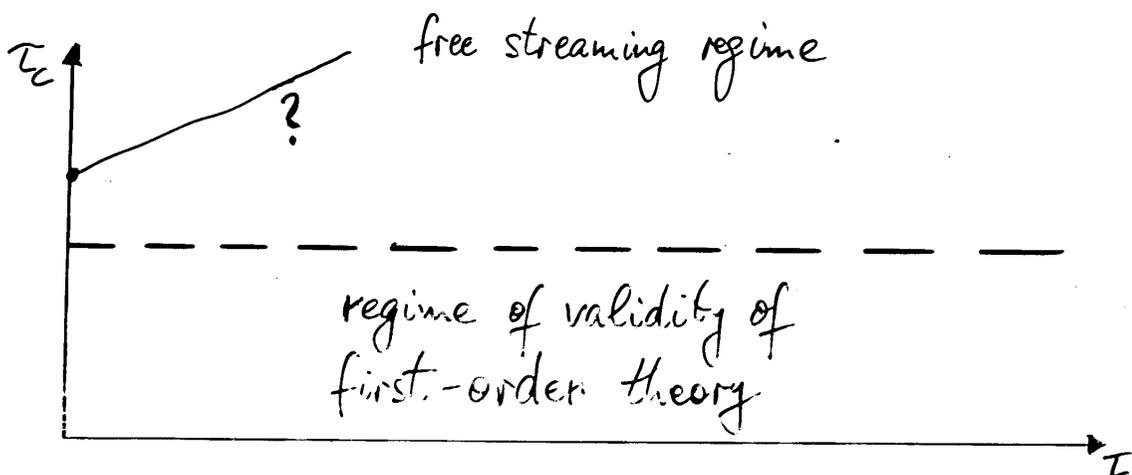
- To obtain small  $R_{II}$  and  $R_0 \approx R_s$  one needs large deviations from Equilibrium already at Hadronization (or else first-order theory applies and  $R_{II}$  doesn't "freeze" ...)



emission duration  $\geq \tau_0$  ;  $R_{||} \sim 2\tau_0$

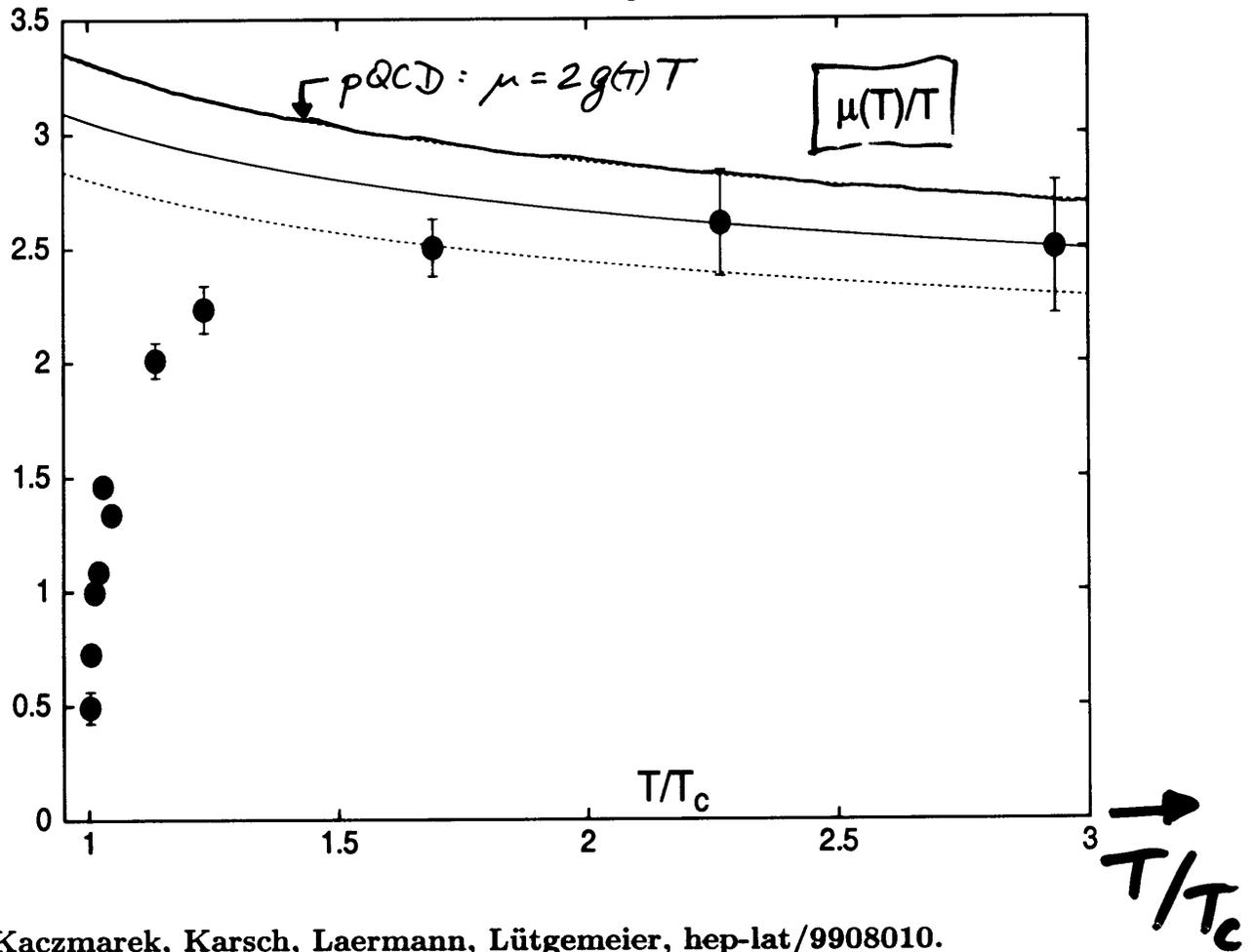
⚡ to data...

→ start with  $\tau_c(\tau_0)$  large,  $\tau_0 < \tau_c$   
 (strongly non-equilibrium hadronization)



# Screening Mass

$$N_f = 0$$



Kaczmarek, Karsch, Laermann, Lütgemeier, hep-lat/9908010.

- Above  $\sim 2T_c$  perturbation theory gives reasonable value for screening mass
- Below  $\sim 2T_c$ , static electric fields seem to be much lighter ...  
→ screening "less effective"

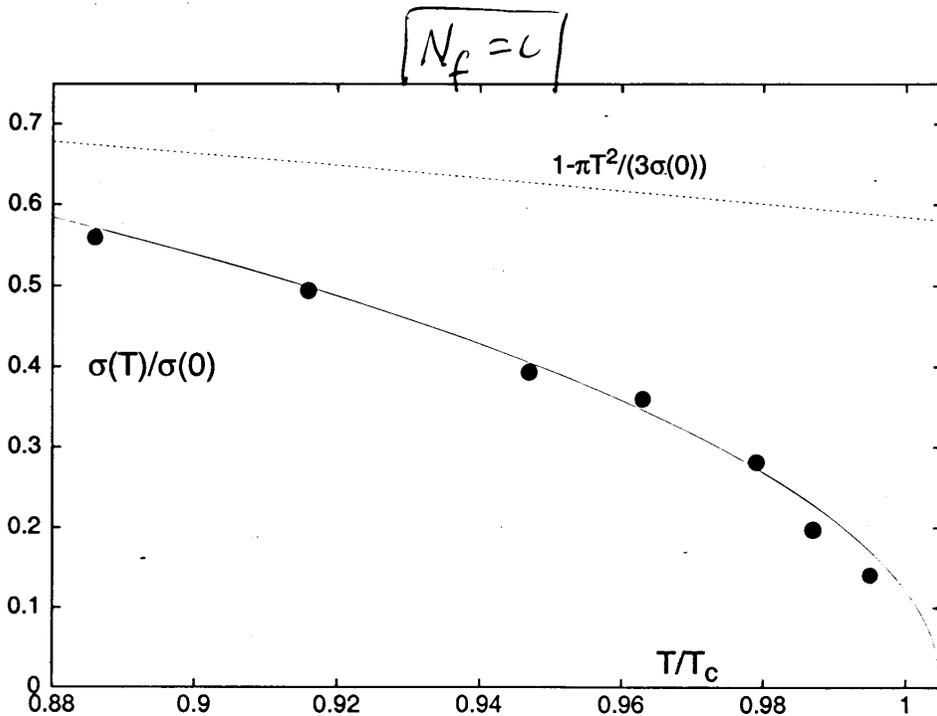
# Rapid increase of the String Tension below $T_c$

$$T < T_c: \quad \boxed{\frac{\sigma(T_c)}{\sigma(0)} \sim \frac{1}{10}}$$

→ Weakly 1st order!  
 (Pisarski calls it nearly 2nd order)

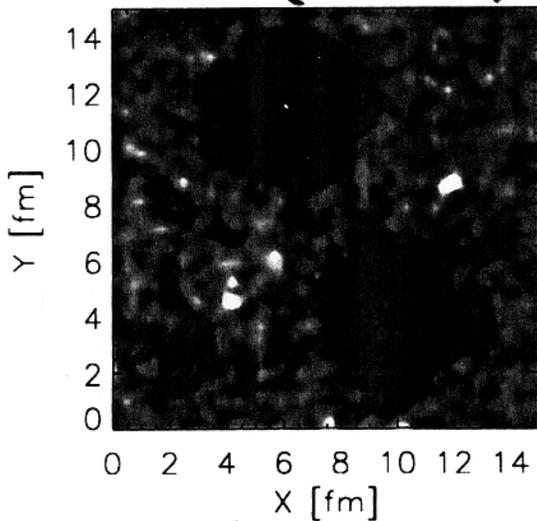
Also,  $m_e = \frac{\sigma(T)}{T}$  is small  $e T_c$

↳ correlation length  $\xi_e = \frac{1}{m_e}$  is "large"!!  
 ( $\sim 1 \text{ fm}$ )



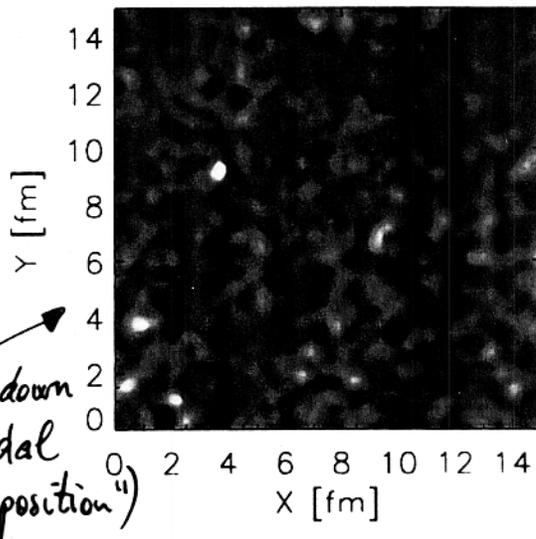
D. Kazmarek et al.,  
 hep-lat/9907010

Slow expansion  
( $\tau \sim 2000 \text{ fm/c}$ )

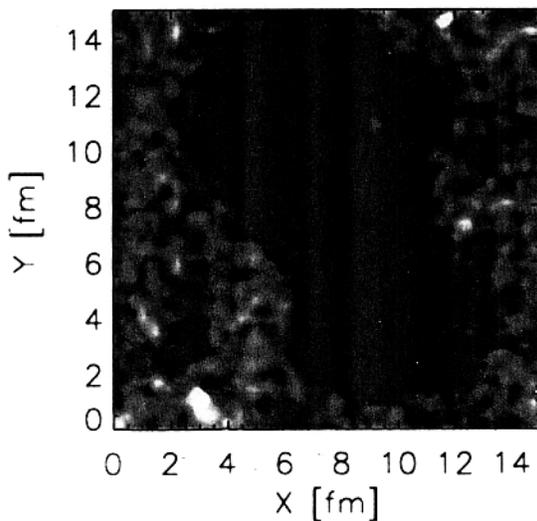


← bubbles form...

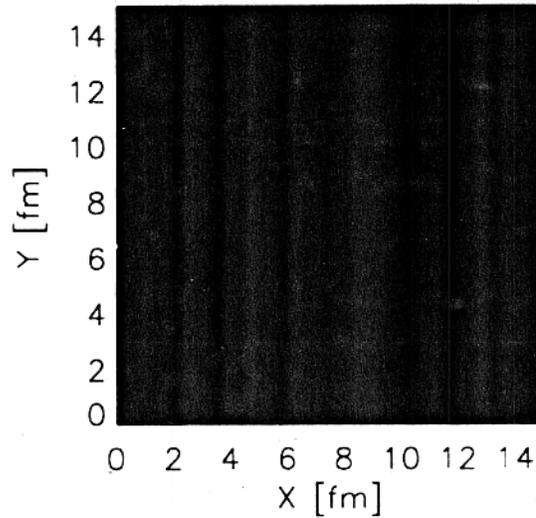
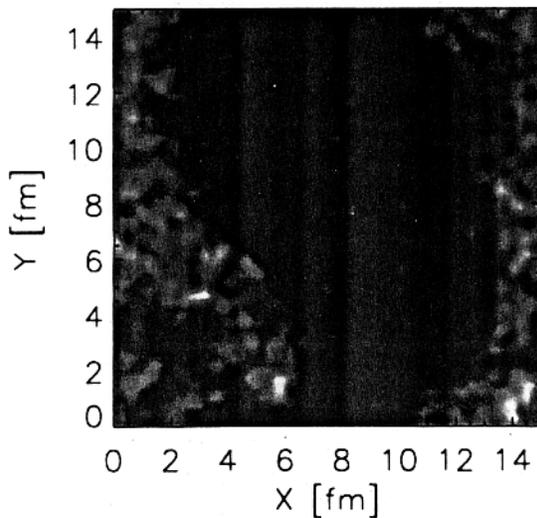
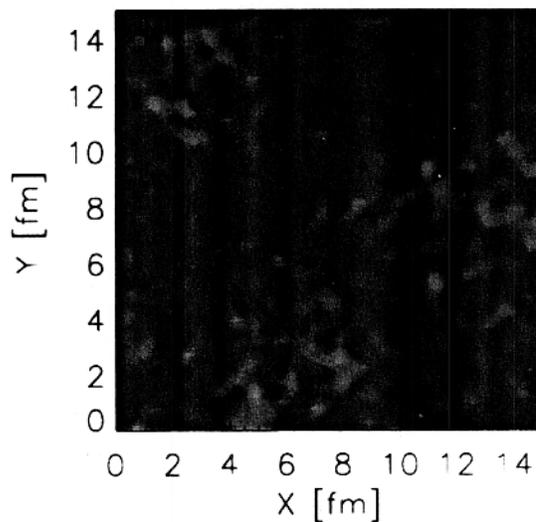
rapid expansion  
( $\tau \sim 100 \text{ fm/c}$ )



roll down  
("spinodal decomposition")

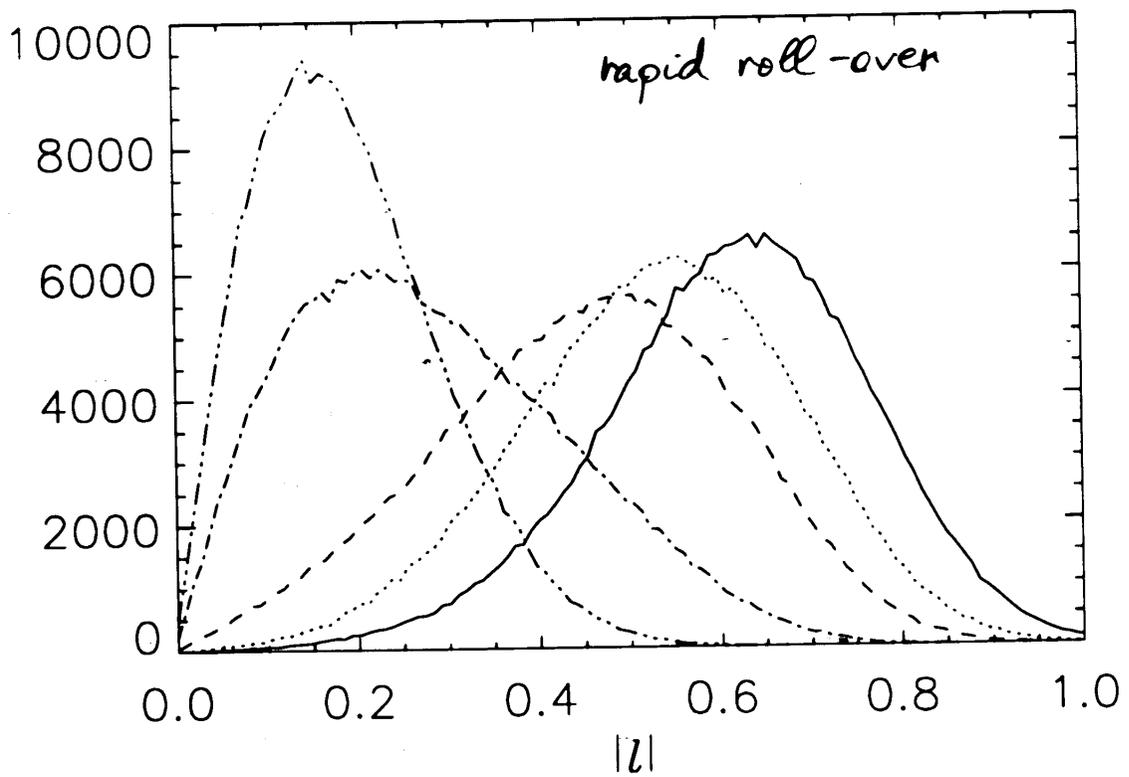
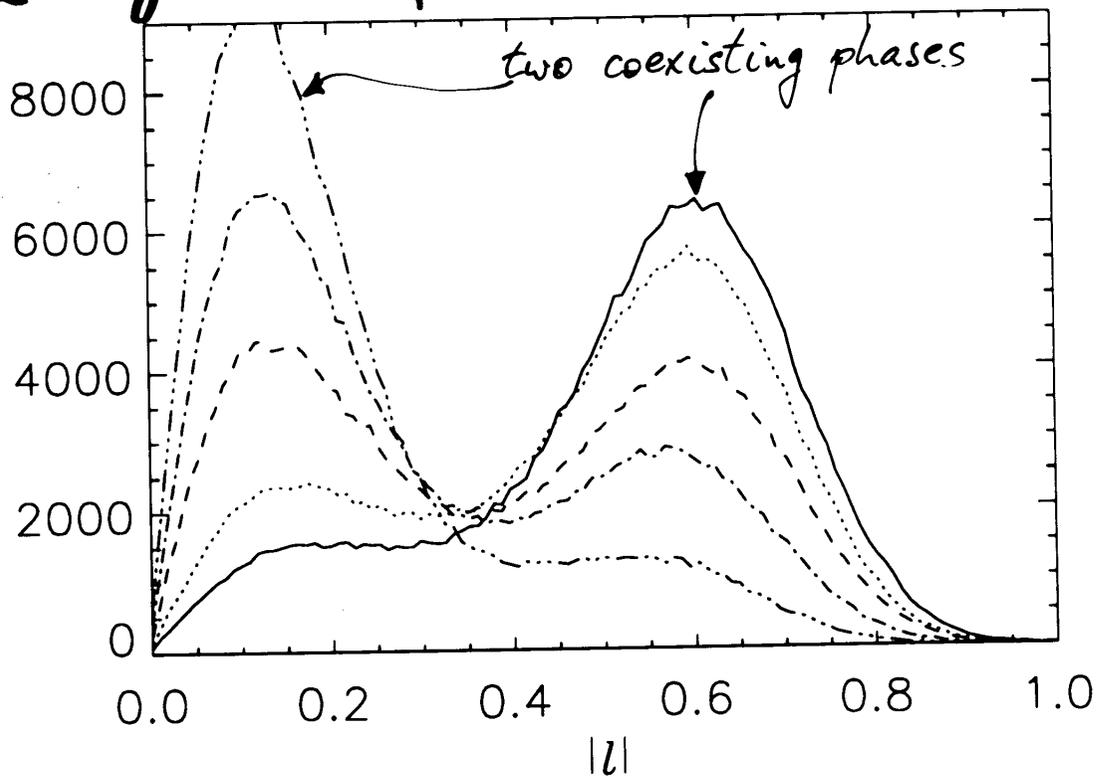


... and coalesce



Ove Scavenius, A.D., Andrew Jackson  
hep-ph/0103219

# Histogram of Field Distribution



# SUMMARY

- For small deviations from local Equilibrium ( $\tau_c$  small) and 1d Bj expansion
  - i) single-inclusive distribution  $\tilde{N}(p)$  can "freeze" if  $\tau_c \sim \tau - z$
  - ii)  $R_{||}$  can not "freeze" for  $\tau_c > 0$
- Thus, to obtain small  $R_{||}$  and  $R_0 \sim R_S$  hadronization into very dissipative hadron fluid (large  $\tau_c$ ) may be required
  - non-equilibrium confinement transition, hadron temperature  $T_0 < T_c$
- This is exactly what has been predicted from the effective Potential for SU(3) Polyakov Loops matched to Lattice Data!

Ove Scavenius, A.D., Andrew Jackson:  
PRL. 87 (2001) 182.302.